The Gambling Effect of Final-Offer Arbitration in Bargaining

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Abstract:

This paper studies an alternating-offer bargaining model in which an arbitrator uses final-offer arbitration to determine the outcome if players fail to reach agreement before a bargaining deadline. We find that if players are sufficiently patient, they will always reject opponents' offers during bargaining, and thus the game always moves to arbitration. Our finding—which is due to the so-called "gambling effect" of final-offer arbitration that we discover—is in sharp contrast to the prediction in the literature that the threat of going to final-offer arbitration encourages players to reach negotiated agreements before arbitration. Our result also holds in various extensions.

Keywords: Bargaining; alternating offer; final-offer arbitration; deadline.
JEL classification: C78; J52

1 Introduction

Arbitration is a common dispute-resolution mechanism. In the US, when bargaining between employees and employers fails, it is sometimes required by law that an arbitration mechanism be used. Two arbitration mechanisms are widely used. One is *conventional arbitration*, in which an arbitrator is allowed to choose any outcome as the arbitration outcome. The other is *final-offer arbitration*, in which the arbitrator can only choose one of the players' offers as the arbitration outcome. Final-offer arbitration was introduced by Stevens (1966), and has been widely used in the public sector in the US to resolve labor disputes. Since final-offer arbitration does not allow a compromised outcome, people tend to believe that the use of final-offer arbitration has the advantage of encouraging players to reach negotiated agreements before arbitration (e.g., Stevens (1966)).

A central question we explore is whether final-offer arbitration indeed encourages players to reach agreements before arbitration. In particular, we will study the role of final-offer arbitration in a model that integrates bargaining and arbitration, and examine how the potential threat of going to final-offer arbitration affects players' strategic behavior in bargaining.

The game analyzed in this paper consists of two stages: a bargaining stage and an arbitration stage. The bargaining stage consists of N periods. Two players make offers alternately in the N periods of the bargaining stage. If, in any period, an offer is accepted, then the game ends immediately with the accepted offer being the outcome. If the offers in all N periods are rejected, then the game moves to the arbitration stage, in which an arbitrator uses *final-offer arbitration* to determine the outcome. In particular, we assume that in the arbitration stage, each player submits all offers that he has ever proposed in the bargaining stage to the arbitrator, and the arbitrator chooses either Player 1's most generous offer or Player 2's most generous offer as the outcome, depending on which offer is closest to the arbitrator's ideal settlement. We also assume that the arbitrator's ideal settlement is the arbitrator's private information, although its distribution is common knowledge among players.

Our model has two key features. First, there is a deadline in bargaining, which is common in practice. For example, in Major League Baseball (MLB), a professional player and his team can bargain over the player's new contract prior to the second week of December (and if an agreement is not reached by then, an arbitrator will be called in to determine the outcome). Second, we assume that, in the arbitration stage, each player submits the entire history of his offers to the arbitrator, and the arbitrator chooses either Player 1's most generous offer or Player 2's most generous offer as the outcome. Equivalently, we can let each player submit the final offer he made in the bargaining stage to the arbitrator, and assume that during bargaining, players can only make concessions when they make new offers (this implies that a player's final offer must also be his most generous offer).

Our main findings are as follows. (i) If players are sufficiently patient, then in any subgame perfect equilibrium (SPE), players' (equilibrium) offers will always be rejected by their opponents, and thus the game will always move to arbitration. (ii) We characterize the unique equilibrium (arbitration) payoff of each player, and find that a player's equilibrium payoff does not depend on whether the player makes the first offer, but rather on whether the player makes the last offer. (iii) Our result in (i) holds in several variations of the basic model, which include allowing the arbitrator's ideal settlement to be influenced by players' offers, allowing players to strategically delay their responses in bargaining, and allowing players to submit new offers when the game moves to arbitration.

The finding that players will never reach agreement before arbitration when players are sufficiently patient is surprising. The intuition is as follows. In final-offer arbitration, the arbitration outcome can only be Player 1's (most generous) offer or Player 2's (most generous) offer. It turns out that this restriction on the possible arbitration outcomes leads players to prefer that the game be moved to the arbitration stage so that they can "gamble" on the arbitration outcome. In particular, suppose that the game now moves to period t of the bargaining stage, and Player j makes an offer in this period. Then for any offer Player j makes (as long as the offer is not extremely generous, which will not occur in equilibrium),

Player i has an incentive to reject the offer. This is because acceptance of Player j's offer will end the game with Player j's offer as the outcome, but rejection of the offer will move the game one step closer to arbitration. Notice that if the game eventually moves to arbitration (which can always be reached if, in all later periods, Player i simply makes extreme offers and rejects all offers made by Player j), the worst case for Player i will be that Player j's most generous offer in all N periods (which must be more favorable to Player i than Player j's offer in period t) is chosen as the arbitration outcome.[1] That is, "gambling" on the arbitration outcome is "riskless." Of course, reaching an outcome in the arbitration stage incurs some time cost. However, as long as players are sufficiently patient, the *gambling effect* of final-offer arbitration will dominate the time cost of delay, and thus players will not reach agreement before arbitration.

Our result contrasts with Stevens (1966), who predicted that the threat of going to final-offer arbitration encourages players to reach negotiated agreements before arbitration. Stevens' prediction is based on the observation that final-offer arbitration can induce players to make moderate demands. This is true, because making an unreasonably high demand will cause it to be chosen with a low probability. However, *making moderate demands does not necessarily mean that players will reach negotiated agreements before arbitration.* Actually, as we illustrate above, the gambling effect of final-offer arbitration leads players to reject opponents' offers, no matter how generous they might be (unless the offers are to give up almost the entire pie).

In this paper, we also characterize players' equilibrium (arbitration) payoffs. We find that whether a player moves first is not a key determinant of the player's equilibrium payoff. Instead, a player's equilibrium payoff depends on whether the player avoids moving last. The reason is as follows. Given that any SPE of the game is such that players will not reach agreement before arbitration, the first mover does not have any advantage, because players cannot avoid the time cost of delay since the game will move to arbitration in any

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[1] The other case is that Player i's most generous offer is chosen. Player i's most generous offer must be more favorable to Player i than Player j's most generous offer in equilibrium.

case, no matter who starts the first offer. Instead, the order of offers in the last two periods plays a key role in players' equilibrium payoffs, because these two periods are closest to the arbitration stage. The player who moves second-to-last servers as a "Stackelberg leader" in the last two periods, and (under some mild conditions) obtains a higher payoff than the case where the player moves last.[2]

We also consider several extensions of our game. In the first extension, the arbitrator's ideal settlement may be influenced by the offers made by players. This situation is likely to arise when, for example, the arbitrator does not have full information about the case he arbitrates, and thus he relies to some extent on the players' offers in making his decision. We find that our main result—that players will not reach agreement before arbitration— is unchanged, because the intuition of the gambling effect still holds. In addition, if the arbitrator is *impartial*, in the sense that he is influenced by the two players' offers to the same extent, then the players' equilibrium payoffs are exactly the same as the situation where the arbitrator's ideal settlement is not at all influenced by players' offers.[3]

In the second extension, we allow players to strategically delay their responses in the sense that when some player makes an offer in some period n, the player's opponent can make a response in *any* period between $n + 1$ and N (a response is either an acceptance or a counteroffer). Suppose that only Player 1 can make the first offer (e.g., only the employer can initiate the bargaining process with the employee by offering a contract to the employee). We find that there is a *deadline effect* in bargaining, in which Player 1 always delays his first offer until period $N - 1$ (or period N if Player 2 is allowed to make a counteroffer in

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[2]Notice that it is meaningless to compare the player who moves second-to-last with the player who moves last, because the arbitrator may be biased toward one of the players. However, we can compare a player's payoff in the game in which the player moves second-to-last with the same player's payoff in the game in which he moves last.

[3]The reason is as follows. If the arbitrator is impartial, then the influence of players' offers on the arbitrator's ideal settlement can only change the extent to which an arbitrator favors a player, and can never change the arbitrator's choice of favored player (this is true even if the influence of players' offers on the arbitrator's ideal settlement is very large). This implies that for any given Player 1's most generous offer and Player 2's most generous offer, the probability that Player 1's most generous offer is chosen and the probability that Player 2's most generous offer is chosen are the same as the basic model in which the arbitrator's ideal settlement is not influenced by players' offers.

the arbitration stage) so that after Player 2 makes a counteroffer, Player 1 is not able to make any futher offer. Therefore, in contrast to Admati and Perry (1987) where delay is used as a signaling device of a player's strength, in our model, Player 1 uses delay to gain an advantageous position so that he can commit not to make the last offer.[4]

In the final extension, we allow players to strategically (and simultaneously) submit new offers in the arbitration stage, and the arbitrator chooses one player's submitted offer as the outcome. Our main result is unchanged as long as it is assumed that players can only make concessions when they make new offers. This is because (i) players will never submit compatible offers in the arbitration stage, and (ii) players are not willing to reach agreement before arbitration because any player's concession made in the bargaining stage will weaken the player's position in arbitration.

1.1 Related Literature

Our paper is closely related to Yildiz (2011) and Rong (2012), who consider a simplified version of the game in this paper. In particular, in Yildiz (2011) and Rong (2012), the bargaining stage only consists of two periods. In addition, in Yildiz (2011) and Rong (2012), the arbitrator's ideal settlement is known to players. More particularly, Yildiz (2011) assumes that the arbitrator's ideal settlement is the Nash bargaining solution outcome, and shows that the unique equilibrium of the final-offer arbitration game is such that Player 1 makes the Rubinstein equilibrium offer (Rubinstein (1982)), which Player 2 accepts. Rong (2012) extends Yildiz (2011) to the general case in which the arbitrator's ideal settlement can be any point on the Pareto frontier of the bargaining set. Rong (2012) finds that the equilibrium of the final-offer arbitration game is either an equilibrium with immediate agreement, or an equilibrium with delayed agreement.[5] While players always reach agreement before

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[4]We also consider a continuous-time version of the above model, in which when a player makes a move at time t, the player's opponent can make a response at *any* time between $t + \epsilon$ and T (where ϵ is the minimal time needed to make a response and T is the bargaining deadline). We find that as the minimal time needed for a response goes to zero, Player 1 will make his first offer exactly at the bargaining deadline.

[5]In particular, Rong (2012) shows that if the arbitrator does not excessively favor one player, then players will reach immediate agreement and the equilibrium outcome coincides with the Rubinstein equilibrium

arbitration in Yildiz (2011) and Rong (2012), it is never the case in our game. That is, the findings of Yildiz (2011) and Rong (2012) are not robust to the introduction of uncertainty regarding the arbitrator's ideal settlement, no matter how small the uncertainty might be.[6]

Our paper differs from most theoretical papers in the arbitration literature in that our model integrates bargaining and arbitration, in which strategic interactions between bargaining and arbitration are allowed. In contrast, the literature mainly focuses on players' behavior and payoffs when the game has already moved to the arbitration stage (see, e.g., Farber (1980), Brams and Merrill (1983), and Deck and Farmer (2007)). The literature usually uses the so-called *contract zone* (i.e., the range of potential settlements that both players consider preferable to arbitration) to characterize players' bargaining behavior. Such characterization, however, is indirect and imprecise.

There are some exceptions in the literature in which the analysis of players' bargaining behavior do not rely on the contract zone. Compte and Jehiel (2004) consider a model in which players make alternating concessions and at any time, the responding player has the option of terminating the negotiation.[7] In addition, this outside option payoff of a player depends on the concessions made by players in negotiation. They found that the mere threat of triggering the outside option forces the equilibrium concessions to be gradual. A key assumption underlying Compte and Jehiel (2004) is that the larger the concession a party has made in negotiation, the higher the outside option payoff of the other party. This assumption is suitable for conventional arbitration (which is often split-the-difference arbitration in practice). However, in final-offer arbitration, a larger concession of a party does not necessarily mean that the arbitration payoff of the other party will increase because the other

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outcome. If the arbitrator sufficiently favors the player making the initial offer, then players will reach delayed agreement in equilibrium. If the arbitrator sufficiently favors the player who makes the second offer, then players will reach immediate agreement.

[6]The gambling effect of final-offer arbitration does not hold when the arbitrator's ideal settlement is known by players. This is because the player who moves last in bargaining can always propose an offer that will be chosen by the arbitrator (with probability one) once the game moves to arbitration. Thus, the player's opponent will accept the offer to avoid the time cost of going to arbitration. However, notice that delay in bargaining may arise (although the arbitration stage will never be reached).

[7]Manzini and Mariotti (2001) proposed a similar alternating-offer model with arbitration. However, in Manzini and Mariotti (2001), the arbitration outcome is *exogenously* given.

party's offer will be chosen with a smaller probability (than the case where the former party makes a smaller concession). Our paper thus complements Compte and Jehiel (2004) by showing that even in final-offer arbitration, players do not have incentive to reach immediate agreement by themselves, which results in delay. The common force behind the delay result for conventional arbitration and final-offer arbitration is that in both arbitration formats, the arbitration outcome "respects" players' offers in the sense that a player's (expected) arbitration payoff always lies in between the two players' offers, which implies that accepting the opponent's offer is always worse than going to arbitration.

This paper is organized as follows. Section 2 studies the equilibrium behavior of the alternating-offer final-offer arbitration game, and Section 3 considers several extensions of the basic model. Concluding remarks are offered in Section 4.

2 Theoretical analysis: The basic model

2.1 The setup

Two players, Player 1 and Player 2, are bargaining over the division of a unit of surplus. The players make alternating offers to their opponents, with one offer per period. If they fail to reach agreement in N periods (where $N \geq 2$), an arbitrator is called in and the arbitrator uses final-offer arbitration to determine the outcome. More precisely, we will study the following *alternating-offer final-offer arbitration game*, which consists of two stages: a bargaining stage and an arbitration stage.

- **Bargaining stage:** There are N periods in the bargaining stage. Player 1 and Player 2 make alternating offers in the N periods, with Player 1 making the first offer. If in any period a player's offer is accepted by the opponent, then the game ends immediately and the accepted offer is the outcome. If all offers in N periods are rejected, then the game moves to the arbitration stage.

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- **Arbitration stage:** Each player submits all the offers that he has ever proposed in the bargaining stage to an arbitrator. The arbitrator uses final-offer arbitration to determine the outcome, in which either Player 1's most generous offer or Player 2's most generous offer is chosen as the outcome, depending on which offer is closest to the arbitrator's ideal settlement.

We use $(x_1^t, 1 - x_1^t)$ to represent Player 1's offer in period t when t is odd, and $(x_2^t, 1 - x_2^t)$ to represent Player 2's offer in period t when t is even. Notice that for $i = 1, 2$, x_i^t always represents the share of Player 1 and $1 - x_i^t$ always represents the share of Player 2. We use $(x_1^{(t)}, 1 - x_1^{(t)})$ to denote Player 1's *most generous offer up to period t*, i.e., $x_1^{(t)} = \min\{x_1^1, x_1^3, ..., x_1^t\}$ if t is odd, and $x_1^{(t)} = \min\{x_1^1, x_1^3, ..., x_1^{t-1}\}$ if t is even. Similarly, we use $(x_2^{(t)}, 1 - x_2^{(t)})$ to denote Player 2's *most generous offer up to period t*, i.e., $x_2^{(t)} = \max\{x_2^2, x_2^4, ..., x_2^t\}$ if t is even, and $x_2^{(t)} = \max\{x_2^2, x_2^4, ..., x_2^{t-1}\}$ if t is odd. When $t = 1$, we assume that $x_2^{(t)} = 0$.

We use $(x^*, 1 - x^*)$, or simply x^*, to denote the arbitrator's *ideal settlement*. In final-offer arbitration, Player 1's most generous offer (i.e., $(x_1^{(N)}, 1 - x_1^{(N)})$) is chosen as the outcome whenever $|x_1^{(N)} - x^*| < |x_2^{(N)} - x^*|$, and Player 2's most generous offer (i.e., $(x_2^{(N)}, 1 - x_2^{(N)})$) is chosen as the outcome whenever $|x_2^{(N)} - x^*| < |x_1^{(N)} - x^*|$. For simplicity, we assume that if the two players' offers are equally close to the arbitrator's ideal settlement (i.e., $|x_1^{(N)} - x^*| = |x_2^{(N)} - x^*|$), then $(x_2^{(N)}, 1 - x_2^{(N)})$ is chosen as the outcome.

The arbitrator's ideal settlement is the arbitrator's private information. The distribution of x^*, however, is common knowledge among players. We use F to denote the cumulative distribution function of x^*. The density function of F is denoted by f. We assume that F has a support of $[0, 1]$, and f is continuous and strictly positive on $[0, 1]$.[8]

Let $U_1(\cdot)$ and $U_2(\cdot)$ be the two players' utility functions. We assume that both U_1 and U_2 are twice differentiable, strictly increasing, and (weakly) concave on $[0, 1]$.

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[8]We make this assumption for simplicity. Our main result (Theorem 1) can be extended to the case in which F has a support of $[a, b] \subseteq [0, 1]$, where $a < b$.

For each player, the payoff obtained in period i in the bargaining stage is subject to a discount of δ^{i-1}, where $\delta \in (0, 1]$ is the players' common discount factor. The payoff obtained in the arbitration stage is subject to a discount of δ^N.

2.2 Analysis

Throughout the paper, we assume the following tie-breaking rule: Whenever a player is indifferent between accepting and rejecting an offer, the player always chooses to accept. Our first result is that it is never optimal for a player to first reject the opponent's offer in some period t, and then, in some later period, make an offer that is more generous (to the opponent) than the offer he previously rejected in period t.

Lemma 1. *For any $\delta \in (0, 1]$ and any $1 \leq t \leq N$, in the alternating-offer final-offer arbitration game, it cannot be a subgame perfect equilibrium outcome that Player 1's most generous offer up to period t yields a smaller payoff for Player 1 than Player 2's most generous offer up to period t (i.e., we cannot have $x_1^{(t)} \leq x_2^{(t)}$ in equilibrium).*

The logic behind Lemma 1 can best be explained for the simple case in which the discount factor is one and players have risk-neutral preferences. Suppose that in some period t_1, a player (say, Player 1) proposes an offer, say $(x_1^{t_1}, 1 - x_1^{t_1})$. Suppose Player 2 rejects the offer and makes an offer, say $(x_2^{t_2}, 1 - x_2^{t_2})$ with $x_2^{t_2} \geq x_1^{t_1}$, in some later period $t_2 > t_1$. Then Player 2 will get at most $1 - x_2^{t_2} \leq 1 - x_1^{t_1}$ in equilibrium (because Player 1 can always choose to accept $(x_2^{t_2}, 1 - x_2^{t_2})$ and thus guarantees a payoff of $x_2^{t_2}$). This implies that Player 2 would be better off if he accepts Player 1's offer in period t_1, which contradicts the fact that Player 2 rejects Player 1's offer in period t_1. Therefore, $x_1^{(t)} \leq x_2^{(t)}$ can never occur in equilibrium.

We have the following main result.

Theorem 1. *There exists a $\delta^* \in (0, 1)$ such that if $\delta \in (\delta^*, 1]$, any subgame perfect equilibrium of the alternating-offer final-offer arbitration game must be such that in any period of*

the bargaining stage, the proposing player makes an offer that is rejected by the responding player, and thus the arbitration service will be used.

Theorem 1 is quite general in that it is true, regardless of the specific distribution of the arbitrator's ideal settlement. The intuition is as follows. Assume that N is even and the discount factor is one. Suppose that the game has already moved to the final period of the bargaining stage, so it is Player 2 who makes an offer. Player 2's offer must be less favorable to Player 1 than Player 1's most generous offer up to period $N-1$ (because otherwise the two players would reach agreement at an earlier time; see also Lemma 1). However, for any such offer that Player 2 makes, it is a dominant strategy for Player 1 to reject it. This is because acceptance of Player 2's offer will simply end the game with Player 2's offer as the outcome, while its rejection will move the game to the arbitration stage, in which, with some probability, the arbitrator chooses Player 1's most generous offer up to period $N-1$ as the outcome (which cannot be worse than Player 2's offer in period N for Player 1), and with the remaining probability, the arbitrator chooses Player 2's most generous offer up to period N as the outcome (which also cannot be worse than Player 2's offer in period N for Player 1). A similar analysis demonstrates that in *any* period, it is optimal for the responding player to reject the proposing player's offer, as long as the offer is not to give up the entire pie (but the proposing player will never propose such a generous offer in equilibrium).

2.3 Equilibrium payoffs

For any $x_1, x_2 \in [0,1]$ with $x_1 \geq x_2$, define $P(x_2|x_1, x_2) = F(\frac{x_1 + x_2}{2})$. That is, $P(x_2|x_1, x_2)$ is the probability that Player 2's offer $(x_2, 1 - x_2)$ is chosen as the arbitration outcome when $(x_1, 1 - x_1)$ and $(x_2, 1 - x_2)$ are the two players' most generous offers. Correspondingly, $1 - P(x_2|x_1, x_2)$ is the probability that Player 1's offer $(x_1, 1 - x_1)$ is chosen as the arbitration outcome. Let $g_1(x_1, x_2) = U_1(x_1)(1 - P(x_2|x_1, x_2)) + U_2(x_2)P(x_2|x_1, x_2)$ and $g_2(x_1, x_2) = U_2(1 - x_1)(1 - P(x_2|x_1, x_2)) + U_2(1 - x_2)P(x_2|x_1, x_2)$ be Player 1's and Player 2's arbitration payoffs, respectively.

We say that the two players' arbitration payoffs are *conflicting* if for any $(x_1, x_2) \in [0,1] \times [0,1]$ and any $(x'_1, x'_2) \in [0,1] \times [0,1]$, $g_1(x'_1, x'_2) > g_1(x_1, x_2)$ implies that $g_2(x'_1, x'_2) < g_2(x_1, x_2)$. Notice that if U_1 and U_2 represent risk-neutral preferences, then the two players' arbitration payoffs are always conflicting, regardless of the distribution of the arbitrator's ideal settlement. Through the remainder of this subsection, we assume that players' utilities and the distribution of the arbitrator's ideal settlement are such that the two players' arbitration payoffs are conflicting.

Let $x_1^* = \underset{x_1 \in [0,1]}{argmax}\, g_1(x_1, x_2^*(x_1))$ where $x_2^*(x_1) = \underset{x_2 \in [0,x_1]}{argmax}\, g_2(x_1, x_2)$. Imagine that there is a *sequential-offer final-offer arbitration game* in which Player 1 and Player 2 propose offers sequentially (where Player 1 makes the first offer, Player 2 makes the second offer, and players do not need to respond to the opponents' offers), and the arbitrator chooses the offer that is closest to his ideal settlement as the final outcome. In the equilibrium of such a game, Player 1 will make the offer $(x_1^*, 1 - x_1^*)$ and Player 2 will make the offer $(x_2^*(x_1^*), 1 - x_2^*(x_1^*))$. Let $U_1^* = g_1(x_1^*, x_2^*(x_1^*))$ and $U_2^* = g_2(x_1^*, x_2^*(x_1^*))$. Similarly, we define $x_2^* = \underset{x_2 \in [0,1]}{argmax}\, g_2(x_1^*(x_2), x_2)$ where $x_1^*(x_2) = \underset{x_1 \in [x_2,1]}{argmax}\, g_1(x_1, x_2)$, and let $U_1^{**} = g_1(x_1^*(x_2^*), x_2^*)$ and $U_2^{**} = g_2(x_1^*(x_2^*), x_2^*)$. It is easy to verify that if the two players' arbitration payoffs are conflicting, then U_1^* and U_2^* are well defined and unique (although x_1^* and $x_2^*(x_1^*)$ may not be unique), and U_1^{**} and U_2^{**} are well defined and unique (although x_2^* and $x_1^*(x_2^*)$ may not be unique).[9] We next show that in our game, if N is even, then any subgame perfect equilibrium outcome must be such that the two players' (non-discounted) arbitration payoffs are U_1^* and U_2^*, respectively (noting that Theorem 1 guarantees that any subgame perfect equilibrium outcome must be such that the arbitration stage is reached), and if N is odd, then any subgame perfect equilibrium outcome must be such that the two players' (non-discounted) arbitration payoffs are U_1^{**} and U_2^{**}, respectively.

Proposition 1.

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[9]The assumption that the two players' arbitration payoffs are conflicting implies that if $g_1(x_1, x_2) = g_1(x'_1, x'_2)$ for some $(x_1, x_2) \in [0,1] \times [0,1]$ and $(x'_1, x'_2) \in [0,1] \times [0,1]$, then it must be true that $g_2(x_1, x_2) = g_2(x'_1, x'_2)$. Therefore, although $x_2^*(x_1)$ may not be unique, the exact choice of $x_2^*(x_1)$ does not matter for Player 1's payoff, which implies that U_1^* (and U_2^*) are well defined. Similarly, U_1^{**} and U_2^{**} are well defined.

(i) If N is even, then the unique subgame perfect equilibrium arbitration payoff is U_1^* for Player 1 and U_2^* for Player 2.

(ii) If N is odd, then the unique subgame perfect equilibrium arbitration payoff is U_1^{**} for Player 1 and U_2^{**} for Player 2.

Although according to Proposition 1, players' equilibrium payoffs are unique, the subgame perfect equilibria of the game may not be unique. Suppose N is even. It can be verified that a subgame perfect equilibrium (outcome) of the game is that Player 1 makes the offer $(x_1^*, 1-x_1^*)$ in any odd period, which Player 2 rejects, and Player 2 makes the offer $(x_2^*(x_1^*), 1-x_2^*(x_1^*))$ in any even period, which Player 1 rejects. Another subgame perfect equilibrium (outcome) is that both players make extreme demands in any period prior to period $N-1$ (which the opponents reject), and then in period $N-1$, Player 1 makes the offer $(x_1^*, 1-x_1^*)$, which Player 2 rejects, and in period N, Player 2 makes the offer $(x_2^*(x_1^*), 1-x_2^*(x_1^*))$, which Player 1 rejects.

Proposition 1 assumes that Player 1 makes the first offer. If Player 2 makes the first offer, then it can be shown that if N is even, the two players' equilibrium (non-discounted) arbitration payoffs are U_1^{**} and U_2^{**}, respectively, and if N is odd, the two players' equilibrium (non-discounted) arbitration payoffs are U_1^* and U_2^* respectively. This result, together with Proposition 1, suggests that a player's equilibrium (non-discounted) arbitration payoff does not depend on whether the player moves first, but whether the player moves last or second-to-last.[10]

We next analyze which player—the last mover or the second-to-last mover—has an advantage? We will not compare the payoffs of Player 1 and Player 2, because the two players' payoffs are largely determined by the degree of bias in the arbitrator's ideal settlement. Instead, we will compare the payoffs of the same player in two scenarios: In one, the player

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[10]In particular, the case in which N is even and Player 2 makes the first offer and the case in which N is odd and Player 1 makes the first offer are equivalent in terms of players' (non-discounted) arbitration payoffs (note that in both cases, Player 1 moves last and Player 2 moves second-to-last). Similarly, the case in which N is odd and Player 2 makes the first offer and the case in which N is even and Player 1 makes the first offer are equivalent (note that in both cases, Player 2 moves last and Player 1 moves second-to-last).

moves second-to-last, and in the other, the player moves last. In particular, let N be fixed, and we will compare Player 1's payoffs across the above two scenarios (i.e., we will compare U_1^* and U_1^{**}).

In general, we may have either $U_1^* \geq U_1^{**}$ or $U_1^* < U_1^{**}$, depending on the exact distribution of the arbitrator's ideal settlement. However, as we will show in the next proposition, for a large class of the distributions of the arbitrator's ideal settlement, we always have $U_1^* \geq U_1^{**}$. That is, the player who moves second-to-last has an advantage in terms of equilibrium arbitration payoff. Let k_1 be the minimum of $\dfrac{U_1'(x)}{U_1(x) - U_1(0)}$ where $x \in [0,1]$, and k_2 be the minimum of $\dfrac{U_2'(y)}{U_2(y) - U_2(0)}$ where $y \in [0,1]$, and let $k = \min\{k_1, k_2\}$ (noting that if players are risk-neutral, then $k = 1$).

Proposition 2. *If* $|\dfrac{d \ln f}{dx}| \leq 4k$ *for any* $x \in [0,1]$, *then we have* $U_1^* \geq U_1^{**}$ *and* $U_2^{**} \geq U_2^*$. *That is, the second-to-last mover has an advantage in arbitration payoff compared to the case in which the player moves last.*

The intuition of Proposition 2 is as follows. Let's first consider an (imaginary) *simultaneous-offer final-offer arbitration* model, in which two players make offers simultaneously and then the arbitrator determines the outcome using final-offer arbitration. One can show that such a model must have one pure-strategy Nash equilibrium if the assumption $|\dfrac{d \ln f}{dx}| \leq 4k$ is satisfied. For simplicity, let's assume that this pure-strategy Nash equilibrium is also unique and is denoted by $((\hat{x}_1, 1 - \hat{x}_1), (\hat{x}_2, 1 - \hat{x}_2))$, and let the two players' equilibrium payoffs be $\hat{U}_1$ and $\hat{U}_2$, respectively. Now, let's go back to the sequential-offer final-offer arbitration game. If Player 1 moves second-to-last (i.e., moves first, since there are only two periods in bargaining), Player 1 can always choose to make the offer $(\hat{x}_1, 1 - \hat{x}_1)$, in which Player 2 must make the offer $(\hat{x}_2, 1 - \hat{x}_2)$ in the last period, because $(\hat{x}_2, 1 - \hat{x}_2)$ is a best response to $(\hat{x}_1, 1 - \hat{x}_1)$. However, Player 1 may do better by proposing some other offer. So, Player 1's payoff U_1^* must be no less than $\hat{U}_1$. On the other hand, we can show that if Player 1 moves last, then Player 2 can obtain a payoff of at least $\hat{U}_2$, and thus Player

1's payoff U_1^{**} is at most $\hat{U}_1$ (using the fact that the two players' arbitration payoffs are conflicting). Therefore, we must have $U_1^* \geq U_1^{**}$. A similar analysis shows that $U_2^{**} \geq U_2^*$.

The analysis above resembles the analysis of the Stackelberg duopoly model, in which there are two firms, and one firm (the Stackelberg leader) chooses its quantity of output first, and the other firm (the Stackelberg follower) chooses its quantity of output second. The Stackelberg leader usually has an advantage in profit. A striking feature of our model is that only the player who is the Stackelberg leader in the last two periods of bargaining has an advantage, and the order of offers in all previous periods does not matter. Intuitively, this is because the offers made before the last two periods are not credible, in the sense that players can always "revise" their offers in the last two periods.

We next show that if players have the same utility function and the arbitrator is *unbiased* in the sense that the distribution of the arbitrator's ideal settlement is symmetric around $1/2$, then it does not matter whether a player moves second-to-last or last.

Corollary 1. *Suppose that $|\dfrac{d \ln f}{dx}| \leq 4k$ for any $x \in [0,1]$. If the arbitrator is unbiased (i.e., the distribution of the arbitrator's ideal settlement is symmetric around $1/2$) and $U_1 = U_2$, then a player obtains the same arbitration payoff no matter whether he moves last or second-to-last.*

2.4   Example

Although Theorem 1 suggests that players' (equilibrium) offers will always be rejected by their opponents, it does not imply that players will make *extreme* offers in equilibrium. Instead, when a player makes an offer, assuming that the player's offer is his most generous offer, there is a trade-off between making a demanding offer (which will bring a larger payoff to the player if the offer is selected by the arbitrator) and making a generous offer (which will make the offer more likely to be selected by the arbitrator).

Next, we consider the situation in which players are risk-neutral and the arbitrator's ideal settlement follows the uniform distribution on $[\dfrac{1}{2} - \epsilon, \dfrac{1}{2} + \epsilon]$ with $\epsilon \in (0, \dfrac{1}{2}]$. For simplicity,

we assume that $N = 2$. We will study how the equilibrium offers of the final-offer arbitration game are determined in such a situation. We will also study how a change in the uncertainty of the arbitrator's ideal settlement affects players' equilibrium offers. We have the following result.

Proposition 3. *Suppose that $N = 2$, $U_1(x_1) = x_1$ and $U_2(1 - x_1) = 1 - x_1$, and x^* follows the uniform distribution on $[\frac{1}{2} - \epsilon, \frac{1}{2} + \epsilon]$ with $\epsilon \in (0, \frac{1}{2}]$. Then there exists a $\hat{\delta} < 1$ such that when $\delta \in (\hat{\delta}, 1]$, the unique subgame perfect equilibrium of the alternating-offer final-offer arbitration game is such that Player 1 makes the offer $(\frac{1}{2} + \epsilon, \frac{1}{2} - \epsilon)$, which Player 2 rejects, and Player 2 makes the counteroffer $(\frac{1}{2} - \epsilon, \frac{1}{2} + \epsilon)$, which Player 1 rejects.*

Proposition 3 demonstrates that when the arbitrator's ideal settlement follows a uniform distribution, the two players' equilibrium offers are the extremes of the uniform distribution (this may not be the case for general distributions). Note that when a player makes a more demanding offer, it has two effects on the player's payoff: a *marginal gain effect*, in the sense that the player will obtain more when the player's offer is chosen by the arbitrator, and a *marginal loss effect*, in the sense that the player's offer will be chosen with a smaller probability. In general, the marginal gain and marginal loss of making a more demanding offer also depend on the other player's offer. The uniform distribution assumption ensures the following two facts. First, the effect of player i's offer on player j's marginal gain of making a more demanding offer exactly offsets the effect of player i's offer on player j's marginal loss of making a more demanding offer. This implies that the optimal decision of player j is irrelevant with respect to player i's offer. Second, for any player, the marginal gain of making a more demanding offer exceeds the corresponding marginal loss if and only if the player's demand is less extreme than the relevant endpoint of the uniform distribution. This implies that the two players will make offers at the extremes of the distribution in equilibrium.

Note that the result in Proposition 3 is based on the assumption that players are risk-neutral. If players are risk-averse, it is possible for players to make less extreme demands in

equilibrium.

Although Stevens (1966) conjectures that uncertainty about the arbitrator's ideal settlement in final-offer arbitration causes players to make more moderate demands during bargaining than they would in conventional arbitration, our example shows that an increase of this uncertainty may cause players' offers to diverge more sharply.[11]

3 Extensions

This section studies several extensions. Its main purpose is to check whether the main result in the basic model—especially the gambling effect of final-offer arbitration—still holds in more realistic settings. We will also study players' bargaining behavior in these new settings.

3.1 Endogenous ideal settlement

This subsection studies a model in which the arbitrator's ideal settlement is influenced by players' offers. This describes the situation in which the arbitrator may not have full information about the case he arbitrates, or is not fully confident about his ideal settlement, and thus he may rely to some degree on players' offers when making his decision.

More precisely, following the model of Farber and Bazerman (1986) (see also Farber (1981), who considers a special case of Farber and Bazerman (1986)), we assume that the arbitrator's ideal settlement is a weighted average of two components: the arbitrator's private ideal settlement (which is exogenously given) and the weighted average of the offers made by the players. In particular, the arbitrator's ideal settlement, denoted by x^{**}, is determined by the equation $x^{**} = \alpha x^* + (1 - \alpha)(\gamma x_1 + (1 - \gamma)x_2)$, where x^* is the arbitrator's private ideal settlement, $\alpha \in (0, 1)$ measures the arbitrator's confidence in his private ideal settlement x^* (so $1 - \alpha$ reflects the influence of players' offers on the arbitrator's ideal settlement), and

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[11]If players are very risk-averse in the example, then it is possible for players to make more moderate demands when uncertainty about the arbitrator's ideal settlement increases.

$\gamma \in (0, 1)$ reflects the relative influence of Player 1's offer on the arbitrator's ideal settlement (or the arbitrator's *partiality* toward Player 1).[12] The arbitrator's private ideal settlement, x^*, is only known to the arbitrator, but its distribution is common knowledge among players. We allow α to be dependent on the players' offers. An example is that α is an increasing function of the difference between the two players' proposals (i.e., $x_1 - x_2$). In this case, as the disagreement between players (i.e., the difference between the players' offers) becomes small, the arbitrator relies more on the players' offers to make his decision. For simplicity, we assume that α is known by players.

We call the model described above the *endogenous ideal settlement model*. We obtain the following result, which shows that the main result obtained in the previous section is robust to the introduction of the endogenous ideal settlement.

Proposition 4. *Consider the endogenous ideal settlement model. When players are sufficiently patient, any subgame perfect equilibrium of the alternating-offer final-offer arbitration game must be such that players will not reach agreement before arbitration, and the arbitration service will be used.*

Proposition 4 holds simply because the intuition of the gambling effect of final-offer arbitration discussed in the previous section still holds in the endogenous ideal settlement model. That is, in the endogenous ideal settlement model, it is still true that in any period of the bargaining stage, a player is always better off rejecting his opponent's offer (and moving the game closer to arbitration) than accepting it, because if the game moves to arbitration, the arbitrator is still restricted to choose between the two players' (most generous) offers. The only change from the basic model is that in final-offer arbitration, for the same combination of players' most generous offers, they are now chosen with a different probability distribution. This change may induce players' equilibrium offers and equilibrium payoffs to be different from the basic model.

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[12]Note that the partiality of an arbitrator (which is described by γ) is different from the (ex ante) *biasedness* of an arbitrator (which is described by the distribution of x^*).

Our next result shows that if the arbitrator is *impartial*, in the sense that $\gamma = 1/2$, then players' bargaining behavior and equilibrium payoffs are exactly the same as the basic model.

Corollary 2. *Consider the endogenous ideal settlement model. Suppose that the arbitrator is impartial and players are sufficiently patient. Then the players' equilibrium arbitration payoffs in the alternating-offer final-offer arbitration game are the same as the case in which the arbitrator's ideal settlement is not influenced by the players' offers.*

In final-offer arbitration, a player's offer will be chosen if and only if the player's offer is closer to the arbitrator's ideal settlement than the other player's offer. If the arbitrator is impartial, then the arbitrator's ideal settlement is a weighted average of the arbitrator's private ideal settlement and the *average* (rather than weighted average) of the offers made by the two players. This implies that if a player's offer is closer to the arbitrator's *private ideal settlement* than the other player's offer, then the former player's offer must also be closer to the arbitrator's *ideal settlement* than the latter player's offer. In other words, if the arbitrator is impartial, then the influence of players' offers on the arbitrator's ideal settlement will only change the extent to which an arbitrator favors a player, but is not sufficient to change the arbitrator's choice of favored player. Therefore, players' bargaining behavior and equilibrium payoffs are the same as the situation in which the arbitrator's ideal settlement is not influenced by players' offers.

3.2   Strategic delay

This subsection analyzes the situation in which a player may strategically delay his response to the opponent's offer (we also allow the player who is required to make the first offer to delay his first offer). Clearly, a player's response is an acceptance or a counteroffer (except in the last period, which we will explain shortly). We still assume that the bargaining stage contains N periods. We assume that an offer can be made at the *beginning* of each period, and the response of the offer can be made only at the *end* of this period or the *end* of

any later period (note that the end of one period is also the beginning of the next period).[13] At the end of the last period, the responding player can only respond with acceptance or rejection (with no counteroffer).

We assume that Player 1 is required to make the first offer (e.g., in MLB, only the team can make the first offer by offering the player a new contract). If no offer is made during the bargaining stage, then both Player 1 and Player 2 obtain a payoff of zero. If only one offer is made in the bargaining stage and it is rejected or the opponent does not make any response to the offer, then in the arbitration stage the player who did not make an offer needs to submit an offer to the arbitrator, and the arbitrator chooses between the only two offers.

The following result shows that when strategic delay is allowed, players will still not reach agreement before arbitration. In addition, there is a *deadline effect* in bargaining— i.e., players will start to make offers only when the deadline for bargaining is approaching.

Proposition 5. *Suppose that* $|\frac{d \ln f}{dx}| \leq 4k$, *players' arbitration payoffs are conflicting, and players are sufficiently patient. Then the subgame perfect equilibrium must be such that the game will move to arbitration. In addition, it is optimal for Player 1 to make his first offer at the beginning of period* $N-1$ *or period* N.

According to the analysis in Section 2, a player obtains a higher payoff if he can avoid making the last offer. So, Player 1 has an incentive to delay making his first offer until period $N-1$ or period N, because if Player 1 does so, then Player 2 has to make a counteroffer— either in the last period of the bargaining stage or in the arbitration stage. More importantly, after Player 2 makes the counteroffer, Player 1 cannot respond to Player 2's counteroffer because either the bargaining deadline has been reached (if Player 2 makes the counteroffer in period N) or the game is in the arbitration stage (if Player 2 makes the offer in the arbitration stage). On the other hand, if Player 1 makes an offer in an earlier period, then

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[13]The assumption that an offer can be made at the end of (or the beginning of) a period (rather than in the middle of a period) is for simplicity. In the continuous time model we will describe shortly, we allow a player to make an offer at any time, as long as a minimum time has passed since the opponent's last offer.

Player 1 cannot commit not to respond to Player 2's counteroffer, and thus Player 1 will be worse off (more precisely, Player 1 cannot avoid making the last offer, because Player 2 can always choose to make the offer $(x_2^*, 1 - x_2^*)$ in period $N - 1$ (possibly through delay), and Player 1 has to respond to such an offer by rejecting it and making the counteroffer $(x_1^*(x_2^*), 1 - x_1^*(x_2^*)))$.

Therefore, unlike the model with incomplete information in which delay is usually used as a signaling device (see, e.g., Admati and Perry (1987)), in our model, a player use delay to gain an advantageous position so that he can commit not to make any further offer in the bargaining stage.

We can get a result similar to Proposition 5 by considering a continuous time version of the strategic delay model described above. In particular, suppose that the total time of the bargaining stage is T. Player 1 makes the first offer, and Player 1 can make his first offer at any time between 0 and T. We use m to denote the minimum time needed for a player to make a response (i.e., acceptance or a counteroffer). Unlike the discrete model, we assume that after a player makes an offer at time t, the opponent can make a response at *any* time between $t + m$ and T. We assume that at time T (i.e., the end of the bargaining stage), if a player makes a response, then the player can only choose acceptance or rejection (i.e., a counteroffer cannot be made at time T). We also assume that if there is only one offer made in the bargaining stage and the game moves to the arbitration stage, then the player who does not make any offer needs to submit an offer to the arbitrator.

Similar to Proposition 5, one can show that it will be optimal for Player 1 to make his first offer at any time between $T - 2m$ and T, so that after Player 2 makes a counteroffer, Player 1 does not have enough time to respond to Player 2's offer. Notice that as m goes to zero, the above interval shrinks to T. We thus have the following result.

Corollary 3. *Suppose that* $|\frac{d \ln f}{dx}| \leq 4k$, *players' arbitration payoffs are conflicting, and players are sufficiently patient. In the continuous-time version of the strategic delay model, as the minimal time needed for a response, m, goes to zero, it will be optimal for Player 1*

to make his first offer at the deadline of the bargaining stage.

3.3 Strategic submission of offers in the arbitration stage

In reality, when the disputed parties move to final-offer arbitration, players may be given one final chance to make offers. This section studies a variant of the basic model, in which when the game moves to arbitration, each player is allowed to make one more offer (simultaneously with the other player). More precisely, the game studied in this section is as follows.

- **Bargaining stage:** There are N periods in the bargaining stage. Player 1 and Player 2 make alternating offers in the N periods, with Player 1 making the first offer. If, in any period, a player's offer is accepted by the opponent, then the game ends immediately and the accepted offer is the outcome. If the offers in all N periods are rejected, then the game moves to the arbitration stage.

- **Arbitration stage:** Players submit (new) offers simultaneously. If the offers are compatible, then the outcome that splits the difference between the two players' offers is implemented. Otherwise, an arbitrator uses final-offer arbitration to determine the outcome, i.e., the arbitrator chooses either Player 1's submitted offer or Player 2's submitted offer as the outcome.

We assume that *whenever players make new offers, they can only make concessions.* In other words, a player's offer made in any period (including the arbitration stage) should be (weakly) more generous to his opponent than any of the player's previous offers. This reflects the convention of real-life bargaining, in which players usually honor the concessions that they made in the past. We find that the main result of the basic model is unchanged.

Proposition 6. *Consider the alternating-offer final-offer arbitration game in which players can strategically submit new offers in the arbitration stage and players can only make*

concessions when making new offers. When players are sufficiently patient, in any subgame perfect equilibrium of the game, it must be true that players will not reach agreement before arbitration and the arbitration service will be used.

Proposition 6 holds for the following reasons. First, note that if the game moves to arbitration, then it is never an equilibrium for players to submit compatible offers. Second, the intuition of the gambling effect of final-offer arbitration still holds. That is, for any offer made by a player (say Player i) in the bargaining stage, acceptance of the offer by Player j will cause the offer to be the outcome, while rejection of the offer will guarantee Player j an outcome that is no worse than the above offer made by Player i. The latter is because Player j can always choose to reject all later offers made by Player i and move the game to arbitration, in which either Player i's offer made in the arbitration stage is chosen or Player j's offer made in the arbitration stage is chosen (noting that Player i's offer made in the arbitration stage must be (weakly) more generous than any previous offer made by Player i in the bargaining stage).

4   Conclusion

In this paper, we study a finite-horizon alternating-offer bargaining model in which the game moves to final-offer arbitration if players fail to reach agreement in a finite period of time. We find that in any equilibrium of the game, players will never reach agreement before arbitration, and thus the arbitration service is always used.[14] This is in contrast to the popular belief that the threat of going to final-offer arbitration encourages players to reach agreement before arbitration. Our result also holds in various extensions, including allowing endogenous ideal settlement, allowing strategic delay in bargaining, and allowing strategic submission of new offers in arbitration.

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[14] When final-offer arbitration is the arbitration mechanism, we usually prefer that players reach agreement by themselves rather than resort to arbitration, because (i) the arbitration process is time-costly, and (ii) if the game moves to final-offer arbitration, then the outcome is usually (ex post) "unfair" because a compromised outcome is not allowed.

It is worthy to note that our result relies on several key assumptions. First, there are no arbitration fees in our model. Second, the players are assumed to be sufficiently patient, i.e., the time cost of delay is small. Third, players can only make concessions when they make new offers (in the third extension of the basic model). If any of these assumptions is violated, our result will not hold. In this sense, our model is not intended to overturn the use of final-offer arbitration in reality, but rather emphasizes how the absence of high arbitration costs and the convention that players need to respect past concessions may undermine the effectiveness of final-offer arbitration in practice.

Appendix: Proofs

Proof of Lemma 1:

Suppose there exists some $1 \leq t \leq N$ such that $x_1^{(t)} \leq x_2^{(t)}$. Let $t_1 = \min\{1 \leq s \leq t : x_1^s = x_1^{(t)}\}$ and $t_2 = \max\{1 \leq s \leq t : x_2^s = x_2^{(t)}\}$. Assume that $t_1 < t_2$ (the proof of the other case (i.e., $t_1 > t_2$) is similar). That is, Player 1 makes the offer $(x_1^{t_1}, 1 - x_1^{t_1}) = (x_1^{(t)}, 1 - x_1^{(t)})$ in period t_1, which Player 2 rejects, and Player 2 makes the offer $(x_2^{t_2}, 1 - x_2^{t_2}) = (x_2^{(t)}, 1 - x_2^{(t)})$ in some later period t_2, where $1 - x_1^{t_1} \geq 1 - x_2^{t_2}$. This cannot be an equilibrium outcome, because (i) if Player 2 chooses to accept Player 1's offer $(x_1^{t_1}, 1 - x_1^{t_1})$ in period t_1, then Player 2's payoff is $\delta^{t_1 - 1} U_2(1 - x_1^{t_1})$, and (ii) if Player 2 rejects Player 1's offer $(x_1^{t_1}, 1 - x_1^{t_1})$ in period t_1 and makes an offer $(x_2^{t_2}, 1 - x_2^{t_2})$ in a later period t_2, then Player 2's payoff must not be greater than $\delta^{t_2 - 1} U_2(1 - x_2^{t_2})$ (because Player 1 can guarantee a payoff of $U_1(x_2^{t_2})$ in period t_2 by accepting the offer $(x_2^{t_2}, 1 - x_2^{t_2})$). Since $\delta^{t_1 - 1} U_2(1 - x_1^{t_1}) \geq \delta^{t_2 - 1} U_2(1 - x_2^{t_2})$, Player 2 should accept Player 1's offer in period t_1.[15] $\square$

Proof of Theorem 1:

We consider the simple case in which both players are perfectly patient, i.e., the discount

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[15]If $\delta^{t_1 - 1} U_2(1 - x_1^{t_1}) = \delta^{t_2 - 1} U_2(1 - x_2^{t_2})$ (which occurs when $(1 - x_1^{t_1}) = 1 - x_2^{t_2}$ and $\delta = 1$), then Player 1 is indifferent between accepting Player 1's offer in period t_1 and rejecting it. In this case, Player 1 chooses acceptance (by the tie-breaking rule).

factor $\delta = 1$. The proof for the more general (but much more complicated) case where δ is less than but is sufficiently close to 1 is put at the end of the Appendix.

Suppose N is even (the proof for the case where N is odd is similar). To gain some intuition of the analysis, suppose that the game now moves to period N. The most generous offer that Player 1 has ever made up to period $N - 1$ is $(x_1^{(N-1)}, 1 - x_1^{(N-1)})$, and the most generous offer that Player 2 has ever made up to period $N - 1$ is $(x_2^{(N-1)}, x_2^{(N-1)})$. Then we must have $x_2^{(N-1)} < x_1^{(N-1)}$ (see Lemma 1). Let the offer made by Player 2 in period N be $(x_2^N, 1 - x_2^N)$. If $x_2^N \geq x_1^{(N-1)}$, then $x_2^{(N)} = \max\{x_2^{(N-1)}, x_2^N\} \geq x_1^{(N-1)} = x_1^{(N)}$. This is a contradiction with Lemma 1. So, we must have $x_2^N < x_1^{(N-1)}$. For Player 1, accepting the offer $(x_2^N, 1 - x_2^N)$ will simply result in $(x_2^N, 1 - x_2^N)$ as the outcome, while rejecting the offer will move the game to arbitration, in which either $(x_1^{(N)}, 1 - x_1^{(N)}) = (x_1^{(N-1)}, 1 - x_1^{(N-1)})$ or $(x_2^{(N)}, 1 - x_2^{(N)})$ will be chosen as the arbitration outcome. So, Player 1 is better off by rejecting Player 2's offer because $x_1^{(N-1)} > x_2^N$ and $x_2^{(N)} = \max\{x_2^{(N-1)}, x_2^N\} \geq x_2^N$.

The above reasoning applies to any previous periods. Suppose the game is now in some period $t < N$. Suppose t is odd (the case that t is even is similar). Notice that $(x_1^{(t-1)}, 1 - x_1^{(t-1)})$ is the most generous offer of Player 1 up to period $t - 1$, and $(x_2^{(t-1)}, 1 - x_2^{(t-1)})$ is the most generous offer of Player 2 up to period $t - 1$.[16] Then, we must have $x_2^{(t-1)} < x_1^{(t-1)}$ by Lemma 1. Let the offer made by Player 1 in period t be $(x_1^t, 1 - x_1^t)$. If $x_1^t \leq x_2^{(t-1)}$ (i.e., $1 - x_1^t \geq 1 - x_2^{(t-1)}$), then $x_1^{(t)} = \min\{x_1^{(t-1)}, x_1^t\} \leq x_2^{(t-1)} = x_2^{(t)}$, which violates Lemma 1. So, we must have $x_1^t > x_2^{(t-1)}$. It is optimal for Player 2 to reject such an offer. This is because the acceptance of the offer will simply result in $(x_1^t, 1 - x_1^t)$ as the outcome, while rejecting the offer will bring Player 2 a payoff that is strictly greater than $1 - x_1^t$, because Player 2 can reject the offer and then makes the offer $(x_2^{(t-1)}, 1 - x_2^{(t-1)})$ in all later periods (either Player 1 accepts the offer in some later period or the game moves to arbitration, in which Player 2's most generous offer $(x_2^{(N)}, 1 - x_2^{(N)})$ (where $x_2^{(N)} = x_2^{(t-1)} < x_1^t$) or Player 1's most generous offer $(x_1^{(N)}, 1 - x_1^{(N)})$ (where $x_1^{(N)} \leq x_1^t$) will be chosen; in either case, Player 2's

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[16]If $t = 1$, then let $x_2^{(t-1)} = 0$ and $x_1^{(t-1)} = 1$.

payoff is greater than $1 - x_1^t$). So, Player 2 is better off rejecting Player 1's offer.

So, we have shown that if $\delta = 1$, then in equilibrium, players will make an offer that will be rejected by the opponents in all N periods. $\square$

Proof of Proposition 1:

Suppose N is even. Player 1 can guarantee a payoff of $\delta^N U_1^*$ because Player 1 can propose x_1^* in any odd period t for $1 \leq t \leq N$ (regardless of Player 2's strategy), and (i) if the game moves to arbitration, Player 1's most generous offer will be $(x_1^*, 1 - x_1^*)$, and Player 1's (non-discounted) arbitration payoff must be no less than U_1^*, and (ii) if the game ends before the arbitration stage, then it must be the case that Player 1 obtains a payoff higher than $\delta^N U_1^*$.

Player 2 can guarantee a payoff of $\delta^N U_2^*$ because Player 2 can propose $(0, 1)$ in any even period t for $1 \leq t \leq N - 2$. To see this point, notice that if Player 2 proposes $(0, 1)$ in all even periods t for $1 \leq t \leq N - 2$, then Player 1's optimal offer in period $N - 1$ must be x_1^*, which Player 2 rejects and proposes $(x_2^*(x_1^*), 1 - x_2^*(x_1^*))$ in period N.

So, we have shown that in any subgame perfect equilibrium outcome, Player 1's payoff should be no less than $\delta^N U_1^*$ and Player 2's payoff should be no less than $\delta^N U_2^*$. According to Theorem 1, all equilibria must involve arbitration. Since the two players' arbitration payoffs are conflicting, the subgame perfect equilibrium (non-discounted) arbitration payoff must be unique and must be U_1^* for Player 1 and U_2^* for Player 2.

The proof for the case where N is odd is similar and is omitted. $\square$

Proof of Proposition 2:

Consider a *simultaneous-offer final-offer arbitration game*, in which two players make offers simultaneously and the arbitrator determines the outcome using final-offer arbitration. We now solve the Nash equilibrium of such a game. Let $\hat{x}_1(x_2)$ be Player 1's best response function and $\hat{x}_2(x_1)$ Player 2's best response function. Then we must have:[17]

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[17]Note that it is never a best response for Player 1 to propose an offer such that $x_1 < x_2$. Similarly, it is never optimal for Player 2 to propose an offer such that $x_2 > x_1$. So, we put the restriction $x_1 \geq x_2$ in the following maximization problems.

$$\hat{x}_1(x_2) = argmax_{x_1 \in [x_2, 1]} U_1(x_1)(1 - F(\frac{x_1 + x_2}{2})) + U_1(x_2)F(\frac{x_1 + x_2}{2}) \qquad (1)$$

and

$$\hat{x}_2(x_1) = argmax_{x_2 \in [0, x_1]} U_2(1 - x_1)(1 - F(\frac{x_1 + x_2}{2})) + U_2(1 - x_2)F(\frac{x_1 + x_2}{2}). \qquad (2)$$

The first-order condition of Equation 1 is $U_1'(x_1)(1 - F(\frac{x_1 + x_2}{2})) - (U_1(x_1) - U_1(x_2))f(\frac{x_1 + x_2}{2})\frac{1}{2} = 0$ and the second-order condition is $U_1''(x_1)(1 - F(\frac{x_1 + x_2}{2})) - U_1'(x_1)f(\frac{x_1 + x_2}{2}) - \frac{1}{4}(U_1(x_1) - U_1(x_2))f'(\frac{x_1 + x_2}{2}) \leq 0$. The second-order condition is satisfied under the assumption that $\frac{d \ln f}{dx} \geq -4k$ (i.e., $\frac{f'}{f} \geq -4k$) (and using the fact that $U_1''(x_1) \leq 0$ and $U_1(x_2) \geq U_1(0)$).

The first-order condition of Equation 2 is $-U_2'(1 - x_2)F(\frac{x_1 + x_2}{2}) + (U_2(1 - x_2) - U_2(1 - x_1))f(\frac{x_1 + x_2}{2})\frac{1}{2} = 0$ and the second-order condition is $U_2''(1 - x_2)F(\frac{x_1 + x_2}{2}) - U_2'(1 - x_2)f(\frac{x_1 + x_2}{2}) + \frac{1}{4}(U_2(1 - x_2) - U_2(1 - x_1))f'(\frac{x_1 + x_2}{2}) \leq 0$. This second-order condition is also satisfied under the assumption that $\frac{d \ln f}{dx} \leq 4k$ (i.e., $\frac{f'}{f} \leq 4k$) (and using the fact that $U_2''(1 - x_2) \leq 0$ and $U_2(1 - x_1) \geq U_2(0)$).

We thus have shown that the first-order conditions of both players are also sufficient. Note that the maximization problems in Equations 1 and 2 have solutions because they are maximization problems of continuous functions in compact sets. So, both the best response function of Player 1 and the best response function of Player 2 are well defined. In addition, since f and U_i' are differentiable, the best response functions of both players are continuous. Using the Brouwer fixed point theorem, it can be shown that the pure-strategy Nash equilibrium exists.

We use $((\hat{x}_1, 1 - \hat{x}_1), (\hat{x}_2, 1 - \hat{x}_2))$ to denote the (pure-strategy) Nash equilibrium that brings Player 1 the highest payoff among all (pure-strategy) Nash equilibria, and we denote the two players' payoffs in this equilibrium $\hat{U}_1$ and $\hat{U}_2$, respectively. Similarly, we use $((\hat{x}_1', 1 - \hat{x}_1'), (\hat{x}_2', 1 - \hat{x}_2'))$ to denote the (pure-strategy) Nash equilibrium that brings Player 2 the

highest payoff among all (pure-strategy) Nash equilibria, and we denote the two players' payoffs in this equilibrium $\hat{U}_1'$ and $\hat{U}_2'$, respectively. Then, we must have $\hat{U}_1' \leq \hat{U}_1$. Now, let's consider the *sequential-offer final-offer arbitration game*, in which the two players make offers sequentially (and do not need to choose acceptance or rejection), and the arbitrator uses final-offer arbitration to determine the outcome. Suppose Player 1 makes the first offer. Then Player 1's equilibrium payoff U_1^* must be at least $\hat{U}_1$ (because Player 1 can at least make the offer $(\hat{x}_1, 1 - \hat{x}_1)$). That is, $U_1^* \geq \hat{U}_1$. Now, suppose Player 1 makes the second offer, then Player 2's equilibrium payoff U_2^{**} must be at least $\hat{U}_2'$, which implies that Player 1's equilibrium payoff U_1^{**} is at most $\hat{U}_1'$ (because the two players' arbitration payoffs are conflicting). Since $\hat{U}_1'$ is not greater than $\hat{U}_1$, we have $U_1^{**} \leq \hat{U}_1$. Therefore, we have shown that $U_1^* \geq U_1^{**}$.

Similarly, we can show that $U_2^{**} \geq U_2^*$. $\qquad\qquad\square$

Proof of Corollary 1:

For simplicity, we assume that players have risk-neutral preferences. In particular, we assume that $U_1(x) = U_2(x) = x$, in which the sum of the two players' arbitration payoffs is always 1.

Suppose Player 2 moves last. For any Player 1's most generous offer up to period $N-1$ (i.e, $(x_1^{(N-1)}, 1 - x_1^{(N-1)})$), Player 2 can always propose a "symmetric" offer $(x_2^N, 1 - x_2^N) = (1 - x_1^{(N-1)}, x_1^{(N-1)})$. Such an offer will guarantee Player 2 an arbitration payoff of $1/2$ in the arbitration stage. So, Player 2's equilibrium arbitration payoff must be no less than $1/2$. But Player 2's equilibrium arbitration payoff cannot be strictly greater than $1/2$. This is because if it is strictly greater than $1/2$, then Player 1's equilibrium arbitration payoff will be strictly less than $1/2$. However, if Player 1 moves last, then Player 1's payoff will also be strictly greater than $1/2$ (due to the symmetry of the problem), and this contradicts the fact that Player 1, as the second-to-last mover, has an advantage in arbitration payoff. So, Player 2's arbitration payoff must be $1/2$, and thus Player 1's arbitration payoff is also $1/2$. Similarly, if Player 2 moves second-to-last, then both players' arbitration payoffs are also

1/2.

The analysis above can be easily extended to the general case where the two players' utility functions are the same. $\square$

Proof of Proposition 3:

For simplicity, we focus on the case in which $\delta = 1$.

Suppose it is in period 2.

If the offer made by Player 1 in period 1 is such that $x_1 \leq \frac{1}{2} - \epsilon$, then Player 2 must accept the offer in equilibrium because the ideal point of the arbitrator can never be less than $\frac{1}{2} - \epsilon$.

Now, suppose the offer that Player 1 makes in period 1 is such that $x_1 > \frac{1}{2} - \epsilon$. It is never optimal for Player 2 to make an offer $(x_2, 1 - x_2)$ with either $\frac{x_1 + x_2}{2} < \frac{1}{2} - \epsilon$ or $\frac{x_1 + x_2}{2} > \frac{1}{2} + \epsilon$.[18] Thus, $(x_2, 1 - x_2)$ must be such that $\frac{1}{2} - \epsilon \leq \frac{x_1 + x_2}{2} \leq \frac{1}{2} + \epsilon$. Also, notice that it is never optimal for Player 2 to make an offer $(x_2, 1 - x_2)$ with $x_2 \geq x_1$. We thus have $x_2 < x_1$. The fact that $\frac{1}{2} - \epsilon \leq \frac{x_1 + x_2}{2} \leq \frac{1}{2} + \epsilon$ and $x_2 < x_1$ implies that Player 1 will reject $(x_2, 1 - x_2)$ in equilibrium. This is because the rejection of Player 2's offer will move the game to the arbitration stage, in which either Player 1's offer $(x_1, 1 - x_1)$ or Player 2's offer $(x_2, 1 - x_2)$ is the outcome, while the acceptance of Player 2's offer will simply result in $(x_2, 1 - x_2)$ as the outcome.

Since $\frac{1}{2} - \epsilon \leq \frac{x_1 + x_2}{2} \leq \frac{1}{2} + \epsilon$, we have $P(x_2 | x_1, x_2) = F(\frac{x_1 + x_2}{2}) = \frac{1}{2\epsilon}[\frac{x_1 + x_2}{2} - (\frac{1}{2} - \epsilon)]$. Let $(x_2^*(x_1), 1 - x_2^*(x_1))$ be the optimal offer of Player 2. Then we must have:

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[18]In the former case, Player 2 will obtain a higher payoff by making the offer $(x_2, 1 - x_2)$ with $x_2 = \frac{1}{2} - \epsilon$. In the latter case, if $x_2 \leq x_1$, Player 2 will obtain a higher payoff by making a slightly more demanding offer; if $x_2 > x_1$, Player 2 will obtain a higher payoff by making the offer $(x_2, 1 - x_2)$ with $x_2 = \frac{1}{2} - \epsilon$.

$$x_2^*(x_1) = \underset{x_2 \in [0,x_1)}{argmax}\{U_2(1-x_1)(1-P(x_2|x_1,x_2)) + U_2(1-x_2)P(x_2|x_1,x_2)\}$$

$$= \underset{x_2 \in [0,x_1)}{argmax}\{(1-x_1)(1-P(x_2|x_1,x_2)) + (1-x_2)P(x_2|x_1,x_2)\}$$

$$= \underset{x_2 \in [0,x_1)}{argmax}\{1-x_1 + P(x_2|x_1,x_2)(x_1-x_2)\}$$

$$= \underset{x_2 \in [0,x_1)}{argmax}\{1-x_1 + \frac{1}{2\epsilon}[\frac{x_1+x_2}{2} - (\frac{1}{2}-\epsilon)](x_1-x_2)\}.$$

It can be verified that

$$x_2^*(x_1) = \begin{cases} \dfrac{1}{2} - \epsilon & if \quad \dfrac{1}{2} - \epsilon < x_1 < \dfrac{1}{2} + 3\epsilon; \\ 1 + 2\epsilon - x_1 & if \quad x_1 \geq \dfrac{1}{2} + 3\epsilon. \end{cases}$$

Now, let's go back to period 1. We have the following three cases.

(i) If Player 1's offer $(x_1, 1-x_1)$ is such that $x_1 \geq \dfrac{1}{2} + 3\epsilon$, then Player 2 will reject the offer. This is because if Player 2 chooses to accept, then Player 2's payoff is $1 - x_1$. If, instead, Player 2 chooses to reject, then Player 2 will offer $x_2^*(x_1) = 1 + 2\epsilon - x_1$ in period 2, which Player 1 rejects in equilibrium, and Player 2's payoff is $1 - x_1 + \dfrac{1}{2\epsilon}[\dfrac{x_1 + x_2^*(x_1)}{2} - (\dfrac{1}{2} - \epsilon)](x_1 - x_2^*(x_1)) = x_1 - 2\epsilon$, which is strictly greater than $1 - x_1$. Player 1's equilibrium payoff is thus $1 - (x_1 - 2\epsilon) = 1 - x_1 + 2\epsilon$. Notice that Player 1's payoff in this case is not greater than $1 - (\dfrac{1}{2} + 3\epsilon) + 2\epsilon = \dfrac{1}{2} - \epsilon$.

(ii) If Player 1's offer $(x_1, 1-x_1)$ is such that $\dfrac{1}{2} - \epsilon < x_1 < \dfrac{1}{2} + 3\epsilon$, then Player 2 will reject the offer. This is because if Player 2 chooses to accept, then his payoff is $1 - x_1$. If, instead, Player 2 chooses to reject, then Player 2 will offer $x_2^*(x_1) = \dfrac{1}{2} - \epsilon$ in period 2, which Player 1 rejects in equilibrium, and Player 2's payoff is $1 - x_1 + \dfrac{1}{2\epsilon}[\dfrac{x_1 + x_2^*(x_1)}{2} - (\dfrac{1}{2} - \epsilon)](x_1 - x_2^*(x_1)) = 1 - x_1 + \dfrac{1}{4\epsilon}(x_1 - \dfrac{1}{2} + \epsilon)^2$, which is strictly greater than $1 - x_1$. Player 1's payoff is thus $x_1 - \dfrac{1}{4\epsilon}(x_1 - \dfrac{1}{2} + \epsilon)^2$. In this case, Player 1's payoff reaches its maximum $\dfrac{1}{2}$ when $x_1 = \dfrac{1}{2} + \epsilon$.

(iii) If Player 1's payoff $(x_1, 1-x_1)$ is such that $x_1 \leq \dfrac{1}{2} - \epsilon$, Player 2 will accept the offer. Player 1's payoff is thus x_1. In this case, Player 1's maximum payoff is $\dfrac{1}{2} - \epsilon$.

Based on the analysis above, we can see that the unique equilibrium is such that Player 1 makes the offer $(x_1, 1-x_1)$ with $x_1 = \dfrac{1}{2} + \epsilon$, which Player 2 rejects, and Player 2 makes

the counteroffer $(x_2, 1 - x_2)$ with $x_2 = \dfrac{1}{2} - \epsilon$, which Player 1 rejects. $\qquad\qquad$ $\square$

Proof of Corollary 2:

Let $P(x_2|x_1, x_2, x^*)$ be the probability that Player 2's offer is chosen as the final-offer arbitration outcome when Player 1's offer is (x_1, y_1) and Player 2's offer is (x_2, y_2) and the arbitrator's ideal settlement does not have an average offer component (i.e., the arbitrator's ideal settlement is x^*). On the other hand, let $P(x_2|x_1, x_2, x^{**})$ be the probability that Player 2's offer is chosen as the final-offer arbitration outcome when Player 1's offer is (x_1, y_1), Player 2's offer is (x_2, y_2), and the arbitrator's ideal settlement has an average offer component (i.e., the arbitrator's ideal settlement is x^{**}). Then, we have $P(x_1|x_1, x_2, x^{**}) = Prob(x^{**} < \dfrac{x_1 + x_2}{2}) = Prob(\alpha x^* + (1-\alpha)\dfrac{x_1 + x_2}{2} < \dfrac{x_1 + x_2}{2}) = Prob(x^* < \dfrac{x_1 + x_2}{2}) = P(x_2|x_1, x_2, x^*)$. That is, the probability that Player 2's offer is chosen when the arbitrator's ideal settlement is x^{**} is exactly the same as the probability that Player 2's offer is chosen when the arbitrator's ideal settlement is x^*. This implies that the equilibrium outcome of the alternating-offer final-offer arbitration game when the arbitrator's ideal settlement is x^{**} should be the same as the final-offer arbitration game when the arbitrator's ideal settlement is x^*. The remaining proof is the same with the proof of Theorem 1 and is omitted. $\qquad$ $\square$

Sketch of Proof of Proposition 5:

If Player 1 makes the first offer at the beginning of period $N - 1$, then Player 1 can guarantee an (arbitration) payoff of U_1^* because Player 1 can make the offer $(x_1^*, 1 - x_1^*)$ at the beginning of period $N - 1$ (and Player 2 must choose to reject and make the counteroffer $(x_2^*(x_1^*), 1 - x_2^*(x_1^*)))$. It can be verified that Player 1 cannot get a higher payoff. This is because if Player 1 makes an offer in a period earlier than period $N - 1$, then Player 2 can always choose to reject it and make the offer $(x_2^*, 1 - x_2^*)$ at the beginning of period $N - 1$ and thus guarantees a payoff of U_2^{**}, which implies that Player 1's payoff in this case must be not greater than U_1^{**}, which is less than U_1^*.

Similarly, we can show that it is optimal for Player 1 to make the offer at the beginning

of period N (and Player 2 will reject such an offer, and in the arbitration stage, Player 2 will submit the offer $(x_2^*(x_1^*), 1 - x_2^*(x_1^*))$). $\square$

Additional Proof of Theorem 1:

We consider the general case where δ is less than but is sufficiently close to one. We focus on the case where $N = 2$. The proof below can be generalized to the case where $N > 2$.

Suppose Player 2 rejects Player 1's offer $(x_1, 1 - x_1)$ at Stage 1 and the game moves to Stage 2. Assume that $x_1 > 0$.[19] Let $(x_2, 1 - x_2)$ be the offer made by Player 2 at Stage 2. Let $g_1(x_1, x_2) = U_1(x_1)(1 - P(x_2|x_1, x_2)) + U_1(x_2)P(x_2|x_1, x_2)$ be Player 1's arbitration payoff when Player 1's offer is $(x_1, 1 - x_1)$ and Player 2's offer is $(x_2, 1 - x_2)$. Let $\epsilon(\delta) \in [0, x_1)$ be such that $U_1(x_1 - \epsilon(\delta)) = \delta g_1(x_1, x_1 - \epsilon(\delta))$. For any $\delta \in (0, 1]$, $\epsilon(\delta)$ exists and is unique because (i) $U_1(x_1 - \epsilon) - \delta g_1(x_1, x_1 - \epsilon) = (U_1(x_1) - U_1(x_1 - \epsilon))(\delta P(x_1 - \epsilon|x_1, x_1 - \epsilon) - 1) + (1 - \delta)U_1(x_1)$ is continuous and strictly decreasing in ϵ, (ii) when $\epsilon = 0$, we have $U_1(x_1 - \epsilon) - \delta g_1(x_1, x_1 - \epsilon) \geq 0$ because $U_1(x_1) \geq \delta g_1(x_1, x_1) = \delta U_1(x_1)$, and (iii) when $\epsilon = x_1$, we have $U_1(x_1 - \epsilon) - \delta g_1(x_1, x_1 - \epsilon) < 0$ because $U_1(0) < \delta g_1(x_1, 0) = \delta(U_1(x_1)(1 - P(0|x_1, 0)) + U_1(0)P(0|x_1, 0))$. In addition, it can be easily verified that $\epsilon(\delta)$ is strictly decreasing in δ and $\epsilon(1) = 0$.

We have the following two cases.

(i) $x_2 \geq x_1 - \epsilon(\delta)$.

In this case, Player 1 must choose to accept Player 2's offer $(x_2, 1 - x_2)$ in equilibrium, and Player 2's maximum (stage-2) payoff is $U_2(x_1 - \epsilon(\delta))$.

(ii) $x_2 < x_1 - \epsilon(\delta)$.

In this case, Player 1 must choose to reject Player 2's offer $(x_2, 1 - x_2)$ in equilibrium. Let $g_2(x_1, x_2) = U_2(1 - x_1)(1 - P(x_2|x_1, x_2)) + U_2(1 - x_2)P(x_2|x_1, x_2)$ be Player 2's arbitration payoff when Player 1's offer is $(x_1, 1 - x_1)$ and Player 2's offer is $(x_2, 1 - x_2)$. Let $x_2^*(x_1, \delta)$ be such that[20]

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[19]Notice that it is never optimal for Player 1 to make an offer $(x_1, 1 - x_1)$ with $x_1 = 0$ at Stage 1.

[20]We slightly abuse the notation here. Note that there are may be multiple $x_2^*(x_1, \delta)$ that attain the maximum. In that case, $x_2^*(x_1, \delta)$ refers to (any) one of them.

$$x_2^*(x_1, \delta) = \underset{x_2 \in [0, x_1 - \epsilon(\delta)]}{argmax} \ g_2(x_1, x_2).$$

Notice that $x_2^*(x_1, \delta)$ is well-defined, because $g_2(x_1, x_2)$ is continuous (this is because f and U_2 are continuous). Notice that $x_2^*(x_1, 1) < x_1$, because $x_1 > 0$ and f is positive on the whole support $[0, 1]$ by assumption. Let $\delta_1^*(x_1)$ be the unique $\delta \in (0, 1)$ such that $x_1 - \epsilon(\delta) = x_2^*(x_1, 1)$. Then, for any $\delta \in (\delta_1^*(x_1), 1]$, we have $x_2^*(x_1, \delta) = x_2^*(x_1, 1)$. Thus, if $\delta \in (\delta_1^*(x_1), 1]$, then Player 2's maximum (stage-2) payoff is $\delta g_2(x_1, x_2^*(x_1, 1))$ in case (ii).

Let $\delta_2^*(x_1)$ be the unique $\delta \in (0, 1)$ such that $U_2(1 - (x_1 - \epsilon(\delta))) = \delta g_2(x_1, x_2^*(x_1, 1))$. If $\delta \in (\max\{\delta_1^*(x_1), \delta_2^*(x_1)\}, 1]$, then $U_2(1 - (x_1 - \epsilon(\delta))) < \delta g_2(x_1, x_2^*(x_1, 1))$ and thus Player 2's optimal action at Stage 2 is to make the offer $(x_2^*(x_1, 1), 1 - x_2^*(x_1, 1))$, which Player 1 chooses to reject in equilibrium.

Now, let's go back to Stage 1. Notice that it is never optimal for Player 1 to make the offer $(x_1, 1 - x_1)$ with $x_1 = 0$, because he can obtain a positive payoff by making any offer $(x_1, 1 - x_1)$ where $x_1 > 0$. For any Player 1's offer $(x_1, 1 - x_1)$ with $x_1 > 0$, if Player 2 accepts the offer, then Player 2's payoff is $U_2(1 - x_1)$. If instead, Player 2 rejects the offer, assuming that $\delta \in (\max\{\delta_1^*(x_1), \delta_2^*(x_1)\}, 1]$, then Player 2 will make the offer $(x_2^*(x_1, 1), 1 - x_2^*(x_1, 1))$ at Stage 2 (which Player 1 rejects), and Player 2's payoff is $\delta^2 g_2(x_1, x_2^*(x_1, 1))$. Let $\delta_3^*(x_1)$ be the unique $\delta \in (0, 1)$ such that $U_2(1 - x_1) = \delta^2 g_2(x_1, x_2^*(x_1, 1))$, then if $\delta \in (\delta_3^*(x_1), 1]$, we have $U_2(1 - x_1) < \delta^2 g_2(x_1, x_2^*(x_1, 1))$. Thus, if $\delta \in (\max\{\delta_1^*(x_1), \delta_2^*(x_1), \delta_3^*(x_1)\}, 1]$, then Player 2's optimal action at Stage 1 is to reject Player 1's offer $(x_1, 1 - x_1)$. Let x_1^* be such that[21]

$$x_1^* = \underset{x_1 \in [0, 1]}{argmax} \ g_1(x_1, x_2^*(x_1, 1)).$$

Suppose x_1^* exists.[22] Notice that if x_1^* exists, then we must have $x_1^* > 0$. Now, choose any $\eta_1 \in (0, 1)$. We can find an $\eta_2 \in (0, x_1^*)$ such that $U_1(x_1^* - \eta_2) < (\eta_1)^2 g_1(x_1^*, x_2^*(x_1^*, 1))$. Let $\delta^* = \max\{\eta_1, \max_{x_1 \in [x_1^* - \eta_2, 1]}\{\max\{\delta_1^*(x_1), \delta_2^*(x_1), \delta_3^*(x_1)\}\}\}$. So, if $\delta \in (\delta^*, 1]$, then (i) it

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[21]Note that there are may be multiple x_1^* that attain the maximum. In that case, x_1^* refers to one of them.

[22]A sufficient condition for x_1^* to exist is that $x_2^*(x_1, 1)$ is continuous in x_1 (notice that we can always select $x_2^*(x_1, 1)$ in a way such that $x_2^*(x_1, 1)$ is continuous). If $x_2^*(x_1, 1)$ is such that x_1^* does not exist, then the subgame perfect equilibrium does not exist for $\delta \in (\delta^*, 1]$, where δ^* will be defined shortly.

is never optimal for Player 1 to make an offer $(x_1, 1 - x_1)$ with $x_1 \in [0, x_1^* - \eta_2]$, and (ii) if Player 1 makes an offer $(x_1, 1 - x_1)$ with $x_1 \in (x_1^* - \eta_2, 1]$, then it is optimal for Player 1 to make the offer $(x_1^*, 1 - x_1^*)$. As a result, if $\delta \in (\delta^*, 1]$, the optimal offer that Player 1 can make is $(x_1^*, 1 - x_1^*)$.

Let $x_2^*(x_1) = x_2^*(x_1, 1)$. Based on the analysis above, when $\delta \in (\delta^*, 1]$, the subgame perfect equilibrium of the final-offer arbitration game is such that at Stage 1, Player 1 makes the offer $(x_1^*, 1 - x_1^*)$, which Player 2 rejects, and at Stage 2, Player 2 makes the offer $(x_2^*(x_1), 1 - x_2^*(x_1))$, which Player 1 rejects.

Similarly, for the general case where $N > 2$, it can be shown that if players are sufficiently patient, then in equilibrium, players will always reject the opponents' equilibrium offers, and the equilibrium outcome is such that (i) if N is even, then $x_1^{(N)} = x_1^*$ and $x_2^{(N)} = x_2^*(x_1)$, and (ii) if N is odd, then $x_2^{(N)} = x_2^*$ and $x_1^{(N)} = x_1^*(x_2)$, where $x_2^* = \underset{x_2 \in [0,1]}{argmax}\, g_2(x_1^*(x_2), x_2)$ and $x_1^*(x_2) = \underset{x_1 \in [x_2,1]}{argmax}\, g_1(x_1, x_2)$ (noting that the calculations of x_1^*, $x_2^*(x_1)$, x_2^*, and $x_1^*(x_2)$ are all independent of the discount factor δ). $\qquad\square$

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