The Gambling Effect of Final-Offer Arbitration in Bargaining

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Abstract:

This paper studies an alternating-offer bargaining model in which if players fail to reach agreement before a bargaining deadline, then an arbitrator uses final-offer arbitration to determine the outcome in the sense that the arbitrator is committed to choose one of the two players' most generous offer (depending on which player's most generous offer is closer to the arbitrator's ideal settlement). We find that if players are sufficiently patient, they will always reject opponents' offers during bargaining, and thus the game always moves to arbitration. Our finding—which is due to the so-called "gambling effect" of final-offer arbitration that we discover—is in sharp contrast to the prediction in the literature that the threat of going to final-offer arbitration encourages players to reach negotiated agreements before arbitration. Our result also holds in various extensions.

Keywords: Bargaining; alternating offer; final-offer arbitration; deadline.
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1 Introduction

Arbitration is a common dispute-resolution mechanism. In the US, when bargaining between employees and employers fails, it is sometimes required by law that an arbitration mechanism be used. Two arbitration mechanisms are widely used. One is *conventional arbitration*, in which an arbitrator is allowed to choose any outcome as the arbitration outcome. The other is *final-offer arbitration*, in which the arbitrator can only choose one player's offer as the arbitration outcome (which means a compromised outcome is not allowed). In practice, conventional arbitration often takes the form of split-the-difference between players' offers, which often causes the chilling effect, i.e., players tend to make extreme offers and thus are unlikely to reach agreement before arbitration. On the other hand, since final-offer arbitration does not allow a compromised outcome, people tend to believe that the use of final-offer arbitration has the advantage of encouraging players to reach negotiated agreements before arbitration (e.g., Stevens (1966)).

A central question we explore is whether final-offer arbitration indeed encourages players to reach agreements before arbitration. In particular, we will study the role of final-offer arbitration in a model that integrates bargaining and arbitration, and examine how the potential threat of going to final-offer arbitration affects players' strategic behavior in bargaining.

The game analyzed in this paper consists of two stages: a bargaining stage and an arbitration stage. The bargaining stage consists of N periods. Two players make offers alternately in the N periods of the bargaining stage. If, in any period, an offer is accepted, then the game ends immediately with the accepted offer being the outcome. If the offers in all N periods are rejected, then the game moves to the arbitration stage, in which an arbitrator uses *final-offer arbitration* to determine the outcome. In particular, we assume that in the arbitration stage, each player submits all offers that he has ever proposed in the bargaining stage to the arbitrator, and the arbitrator chooses either Player 1's most generous offer or Player 2's most generous offer as the outcome, depending on which offer is closest to the arbitrator's ideal settlement. We also assume that the arbitrator's ideal settlement is the arbitrator's private information, although its distribution is common knowledge among players.

Our model has two key features. First, there is a deadline in bargaining, which is common in practice. For example, in Major League Baseball (MLB), a professional player and his team can bargain over the player's new contract prior to the second week of December (and if an agreement is not reached by then, an arbitrator will be called in to determine the outcome). Second, we assume that, in the arbitration stage, each player submits the entire history of his offers to the arbitrator, and the arbitrator chooses either Player 1's most generous offer or Player 2's most generous offer as the outcome. Equivalently, we can let each player submit the final offer he made in the bargaining stage to the arbitrator, and assume that during bargaining, players can only make concessions when they make new offers (this implies that a player's final offer must also be his most generous offer).

Our main findings are as follows. (i) If players are sufficiently patient, then in any subgame perfect equilibrium (SPE), players' (equilibrium) offers will always be rejected by their opponents, and thus the game will always move to arbitration. (ii) We characterize the unique equilibrium (arbitration) payoff of each player, and find that a player's equilibrium payoff does not depend on whether the player makes the first offer, but rather on whether the player makes the last offer. (iii) Our result in (i) holds in several variations of the basic model, which include allowing the arbitrator's ideal settlement to be influenced by players' offers, allowing players to strategically delay their responses in bargaining, and allowing players to submit new offers when the game moves to arbitration.

The finding that players will never reach agreement before arbitration when players are sufficiently patient is surprising. The intuition can be best explained for the simple case where $N = 2$. In particular, suppose the game now moves to period 2 of the bargaining stage (which is also the final period in bargaining), then for *any* given Player 2's offer made in period 2 (which must be less favorable to Player 1 than Player 1's offer in period 1), it is optimal for Player 1 to reject Player 2's offer as long as Player 1 is sufficiently patient.

This is because rejection of Player 2's offer will move the game to the arbitration stage in which Player 1's offer will be chosen with some positive probability and Player 2's offer will be chosen with the remaining probability (assuming that the distribution of the arbitrator's ideal settlement has full support), while acceptance of Player 2's offer will simply end the game with Player 2's offer as the outcome.[1] It can also be shown that in period 1, for *any* given Player 1's offer, it is optimal for Player 2 to reject Player 1's offer (as long as Player 1's offer is not to give up the entire pie, which will not occur in equilibrium). This is because (i) by rejecting Player 1's offer, Player 2 can make an offer (in period 2) that is more favorable to Player 2 than Player 1's offer, and Player 1 will reject such an offer and thus the game will move to the arbitration stage in which Player 2's offer will be chosen with some positive probability (and Player 1's offer will be chosen with the remaining probability); (ii) by accepting Player 1's offer, the outcome will simply be Player 1's offer; and (iii) for Player 2, option (i) is better than option (ii) as long as Player 2 is sufficiently patient.[2]

In final-offer arbitration, the arbitration outcome can only be Player 1's (most generous) offer or Player 2's (most generous) offer. It is this restriction on the possible arbitration outcomes that leads players to prefer that the game be moved to the arbitration stage so that they can "gamble" on the arbitration outcome. In particular, "gambling" on the arbitration outcome is "riskless" in the sense that when the game moves to arbitration, the worst case is that the opponent's (most generous) offer is chosen. Of course, reaching an outcome in the arbitration stage incurs some time cost. However, as long as players are sufficiently patient, the *gambling effect* of final-offer arbitration will dominate the time cost of delay, and thus players will not reach agreement before arbitration.

Our result contrasts with Stevens (1966), who predicted that the threat of going to

[1] Note that Player 1's offer (which is made in period 1) must be more favorable to Player 1 than Player 2's offer (which is made in period 2)—otherwise, it will be meaningless for Player 2 to reject Player 1's offer and the game will not move to period 2.

[2] Although any offers made by Player 1 and Player 2 will be rejected by their opponents, this does not mean that the players will make extreme offers in equilibrium. In general, there is a tradeoff between making a demanding offer and making a generous offer because a generous offer will be chosen by the arbitrator with a higher probability than a demanding offer.

final-offer arbitration encourages players to reach negotiated agreements before arbitration. Stevens' prediction is based on the observation that final-offer arbitration can induce players to make moderate demands. This is true, because making an unreasonably high demand will cause it to be chosen with a low probability. However, *making moderate demands does not necessarily mean that players will make compatible demands.* Actually, as we illustrate above, the gambling effect of final-offer arbitration leads players to reject opponents' offers, no matter how generous they might be (unless the offers are to give up almost the entire pie).

In this paper, we also characterize players' equilibrium (arbitration) payoffs. We find that whether a player moves first is not a key determinant of the player's equilibrium payoff. Instead, a player's equilibrium payoff depends on whether the player moves last. The reason is as follows. Given that any SPE of the game is such that players will not reach agreement before arbitration, the first mover does not have any advantage, because players cannot avoid the time cost of delay since the game will move to arbitration in any case, no matter who starts the first offer. Instead, the order of offers in the last two periods plays a key role in players' equilibrium payoffs, because these two periods are closest to the arbitration stage. The player who moves second-to-last servers as a "Stackelberg leader" in the last two periods, and (under some mild conditions) obtains a higher payoff than the case where the player moves last.[3]

We also consider several extensions of our game. In the first extension, the arbitrator's ideal settlement may be influenced by the offers made by players. This situation is likely to arise when, for example, the arbitrator does not have full information about the case he arbitrates, and thus he relies to some extent on the players' offers in making his decision. We find that our main result—that players will not reach agreement before arbitration—is unchanged, because the intuition of the gambling effect still holds. In addition, if the

[3]Notice that it is meaningless to compare the player who moves second-to-last with the player who moves last, because the arbitrator may be biased toward one of the players. However, we can compare a player's payoff in the game in which the player moves second-to-last with the same player's payoff in the game in which he moves last.

arbitrator is *impartial*, in the sense that he is influenced by the two players' offers to the same extent, then the players' equilibrium payoffs are exactly the same as the situation where the arbitrator's ideal settlement is not at all influenced by players' offers.

In the second extension, we allow players to strategically delay their responses in the sense that when some player makes an offer in some period n, the player's opponent can make a response in *any* period between $n+1$ and N (a response is either an acceptance or a counteroffer). Suppose that only Player 1 can make the first offer (e.g., only the employer can initiate the bargaining process with the employee by offering a contract to the employee). We find that there is a *deadline effect* in bargaining, in which Player 1 always delays his first offer until period $N-1$ (or period N if Player 2 is allowed to make a counteroffer in the arbitration stage) so that after Player 2 makes a counteroffer, Player 1 is not able to make any further offer. Therefore, Player 1 uses delay to gain an advantageous position so that he can commit not to make the last offer.

In the second extension, we also consider a continuous-time version, in which when a player makes a move at time t, the player's opponent can make a response at *any* time between $t+\epsilon$ and T (where ϵ is the minimal time needed to make a response and T is the bargaining deadline). We find that as the minimal time needed for a response goes to zero, Player 1 will make his first offer exactly at the bargaining deadline.

In the final extension, we allow players to strategically (and simultaneously) submit new offers in the arbitration stage, and the arbitrator chooses one player's submitted offer as the outcome. Our main result is unchanged as long as it is assumed that players can only make concessions when they make new offers. This is because (i) players will never submit compatible offers in the arbitration stage, and (ii) players are not willing to reach agreement before arbitration because any player's concession made in the bargaining stage will weaken the player's position in arbitration.

1.1 Related Literature

Our paper is closely related to Yildiz (2011) and Rong (2012). Both our model and the models in Yildiz (2011) and Rong (2012) consider final-offer arbitration, i.e., all these papers assume that when the game moves to arbitration, the arbitrator chooses (among the two players' most generous offers) the offer that is closest to the arbitrator's ideal settlement as the outcome (noting that in Yildiz (2011) and Rong (2012), there are only 2 periods in the bargaining stage, so a player's most generous offer is simply the only offer that the player makes). A key difference between our model and the literature is that in our model, the arbitrator's ideal settlement is the arbitrator's private information, while in Yildiz (2011) and Rong (2012), the arbitrator's ideal settlement is known to players. In particular, Yildiz (2011) assumes that the arbitrator's ideal settlement is the Nash bargaining solution outcome, and shows that the unique equilibrium of the final-offer arbitration game is such that Player 1 makes the Rubinstein equilibrium offer (Rubinstein (1982)), which Player 2 accepts. Rong (2012) extends Yildiz (2011) to the general case in which the arbitrator's ideal settlement can be any point on the Pareto frontier of the bargaining set. Rong (2012) finds that the equilibrium of the final-offer arbitration game is either an equilibrium with immediate agreement, or an equilibrium with delayed agreement.[4] So, although players always reach agreement before arbitration in Yildiz (2011) and Rong (2012), it is not the case in our game (when players are sufficiently patient).

Our paper differs from most theoretical papers in the arbitration literature in that our model integrates bargaining and arbitration, in which strategic interactions between bargaining and arbitration are allowed. In contrast, the literature mainly focuses on players' behavior and payoffs when the game has already moved to the arbitration stage (see, e.g., Farber (1980), Brams and Merrill (1983), and Deck and Farmer (2007)). The literature

[4] In particular, Rong (2012) shows that if the arbitrator does not excessively favor one player, then players will reach immediate agreement and the equilibrium outcome coincides with the Rubinstein equilibrium outcome. If the arbitrator sufficiently favors the player making the initial offer, then players will reach delayed agreement in equilibrium. If the arbitrator sufficiently favors the player who makes the second offer, then players will reach immediate agreement.

usually uses the so-called *contract zone* (i.e., the range of potential settlements that both players consider preferable to arbitration) to characterize players' bargaining behavior. Such characterization, however, is indirect and imprecise.[5]

There are some exceptions in the literature in which the analysis of players' bargaining behavior do not rely on the contract zone. Compte and Jehiel (2004) consider a model in which players make alternating concessions and at any time, the responding player has the option of terminating the negotiation.[6] In addition, this outside option payoff of a player depends on the concessions made by players in negotiation. They found that the mere threat of triggering the outside option forces the equilibrium concessions to be gradual. A key assumption underlying Compte and Jehiel (2004) is that the larger the concession a party has made in negotiation, the higher the outside option payoff of the other party. This assumption is suitable for conventional arbitration (which is often split-the-difference arbitration in practice). However, in final-offer arbitration, a larger concession of a party does not necessarily mean that the arbitration payoff of the other party will increase because the other party's offer will be chosen with a smaller probability (than the case where the former party makes a smaller concession). Our paper thus complements Compte and Jehiel (2004) by showing that even in final-offer arbitration, players do not have incentive to reach immediate agreement by themselves, which results in delay. The common force behind the delay result for conventional arbitration and final-offer arbitration is that in both arbitration formats, the arbitration outcome "respects" players' offers in the sense that a player's (expected) arbitration payoff always lies in between the two players' offers, which implies that accepting the opponent's offer is always worse than going to arbitration.

This paper is organized as follows. Section 2 studies the equilibrium behavior of the alternating-offer final-offer arbitration game, and Section 3 considers several extensions of

[5]In particular, the contract zone is a range of potential settlements between players, so there is no prediction for which settlement will be the final settlement. Moreover, the use of contract zones ignores the underlying bargaining process.

[6]Manzini and Mariotti (2001) proposed a similar alternating-offer model with arbitration. However, in Manzini and Mariotti (2001), the arbitration outcome is *exogenously* given.

the basic model. Section 4 discusses some other arbitration formats where the arbitrator is not committed to select one of the two players' most generous offer. Concluding remarks are offered in Section 5.

2 Theoretical analysis: The basic model

2.1 The setup

Two players, Player 1 and Player 2, are bargaining over the division of a unit of surplus. The players make alternating offers to their opponents, with one offer per period. If they fail to reach agreement in N periods (where $N \geq 2$), an arbitrator is called in and the arbitrator uses final-offer arbitration to determine the outcome. More precisely, we will study the following *alternating-offer final-offer arbitration game*, which consists of two stages: a bargaining stage and an arbitration stage.

- **Bargaining stage:** There are N periods in the bargaining stage. Player 1 and Player 2 make alternating offers in the N periods, with Player 1 making the first offer. If in any period a player's offer is accepted by the opponent, then the game ends immediately and the accepted offer is the outcome. If all offers in N periods are rejected, then the game moves to the arbitration stage.

- **Arbitration stage:** Each player submits all the offers that he has ever proposed in the bargaining stage to an arbitrator. The arbitrator uses final-offer arbitration to determine the outcome, in which either Player 1's most generous offer or Player 2's most generous offer is chosen as the outcome, depending on which offer is closest to the arbitrator's ideal settlement.

We use $(x_1^t, 1-x_1^t)$ to represent Player 1's offer in period t when t is odd, and $(x_2^t, 1-x_2^t)$ to represent Player 2's offer in period t when t is even. Notice that for $i = 1, 2$, x_i^t always represents the share of Player 1 and $1 - x_i^t$ always represents the share of Player

2. We use $(x_1^{(t)}, 1 - x_1^{(t)})$ to denote Player 1's *most generous offer up to period* t, i.e., $x_1^{(t)} = \min\{x_1^1, x_1^3, ..., x_1^t\}$ if t is odd, and $x_1^{(t)} = \min\{x_1^1, x_1^3, ..., x_1^{t-1}\}$ if t is even. Similarly, we use $(x_2^{(t)}, 1 - x_2^{(t)})$ to denote Player 2's *most generous offer up to period* t, i.e., $x_2^{(t)} = \max\{x_2^2, x_2^4, ..., x_2^t\}$ if t is even, and $x_2^{(t)} = \max\{x_2^2, x_2^4, ..., x_2^{t-1}\}$ if t is odd. When $t = 1$, we assume that $x_2^{(t)} = 0$.

We use $(x^*, 1 - x^*)$, or simply x^*, to denote the arbitrator's *ideal settlement*. In final-offer arbitration, Player 1's most generous offer (i.e., $(x_1^{(N)}, 1 - x_1^{(N)})$) is chosen as the outcome whenever $|x_1^{(N)} - x^*| < |x_2^{(N)} - x^*|$, and Player 2's most generous offer (i.e., $(x_2^{(N)}, 1 - x_2^{(N)})$) is chosen as the outcome whenever $|x_2^{(N)} - x^*| < |x_1^{(N)} - x^*|$. For simplicity, we assume that if the two players' offers are equally close to the arbitrator's ideal settlement (i.e., $|x_1^{(N)} - x^*| = |x_2^{(N)} - x^*|$), then $(x_2^{(N)}, 1 - x_2^{(N)})$ is chosen as the outcome.

The arbitrator's ideal settlement is the arbitrator's private information. The distribution of x^*, however, is common knowledge among players. We use F to denote the cumulative distribution function of x^*. The density function of F is denoted by f. We assume that F has a support of $[0, 1]$, and f is continuous and strictly positive on $[0, 1]$.[7]

Let $U_1(\cdot)$ and $U_2(\cdot)$ be the two players' utility functions. We assume that both U_1 and U_2 are twice differentiable, strictly increasing, and (weakly) concave on $[0, 1]$.

For each player, the payoff obtained in period i in the bargaining stage is subject to a discount of δ^{i-1}, where $\delta \in (0, 1]$ is the players' common discount factor. The payoff obtained in the arbitration stage is subject to a discount of δ^N.

2.2 Analysis

Throughout the paper, we assume the following tie-breaking rule: Whenever a player is indifferent between accepting and rejecting an offer, the player always chooses to accept. Our first result is that it is never optimal for a player to first reject the opponent's offer in

[7]We make this assumption for simplicity. Our main result (Theorem 1) can be extended to the case in which F has a support of $[a, b] \subseteq [0, 1]$, where $a < b$.

some period t, and then, in some later period, make an offer that is more generous (to the opponent) than the offer he previously rejected in period t.

Lemma 1. *For any $\delta \in (0,1]$ and any $1 \leq t \leq N$, in the alternating-offer final-offer arbitration game, it cannot be a subgame perfect equilibrium outcome that Player 1's most generous offer up to period t yields a smaller payoff for Player 1 than Player 2's most generous offer up to period t (i.e., we cannot have $x_1^{(t)} \leq x_2^{(t)}$ in equilibrium).*

The logic behind Lemma 1 can best be explained for the simple case in which the discount factor is one and players have risk-neutral preferences. Suppose that in some period t_1, a player (say, Player 1) proposes an offer, say $(x_1^{t_1}, 1 - x_1^{t_1})$. Suppose Player 2 rejects the offer and makes an offer, say $(x_2^{t_2}, 1 - x_2^{t_2})$ with $x_2^{t_2} \geq x_1^{t_1}$, in some later period $t_2 > t_1$. Then Player 2 will get at most $1 - x_2^{t_2} \leq 1 - x_1^{t_1}$ in equilibrium (because Player 1 can always choose to accept $(x_2^{t_2}, 1 - x_2^{t_2})$ and thus guarantees a payoff of $x_2^{t_2}$). This implies that Player 2 would be better off if he accepts Player 1's offer in period t_1, which contradicts the fact that Player 2 rejects Player 1's offer in period t_1. Therefore, $x_1^{(t)} \leq x_2^{(t)}$ can never occur in equilibrium.

We have the following main result.

Theorem 1. *There exists a $\delta^* \in (0,1)$ such that if $\delta \in (\delta^*, 1]$, any subgame perfect equilibrium of the alternating-offer final-offer arbitration game must be such that in any period of the bargaining stage, the proposing player makes an offer that is rejected by the responding player, and thus the arbitration service will be used.*

Theorem 1 is quite general in that it is true, regardless of the specific distribution of the arbitrator's ideal settlement. The intuition of Theorem 1 can be explained as follows. For simplicity, assume that the discount factor is one. Suppose that the game now moves to period t of the bargaining stage, and it is Player j who makes an offer in this period. Then for any offer Player j makes, it is a dominant strategy for Player i to reject the offer. This is because acceptance of Player j's offer will end the game with Player j's offer as the outcome, but rejection of the offer will move the game one step closer to arbitration. Notice that if the game eventually moves to arbitration (which can always be reached if, in all later periods, Player i simply makes extreme offers and rejects all offers made by Player j), there are only two possibilities. One is that Player j's most generous offer in all N periods is chosen as the arbitration outcome. The other is that Player i's most generous offer in all N periods is chosen. In both possibilities, the arbitration outcome is more favorable to Player i than Player j's offer in period t, because (i) Player j's most generous offer in all N periods must be more favorable to Player i than Player j's offer in period t, and (ii) Player i's most generous offer in all N periods must be more favorable to Player i than Player j's most generous offer in all N periods (see also Lemma 1). Finally, the above intuition also goes through if the discount factor is less than but sufficiently close to one.

It is worthwhile to note that delay in equilibrium within the framework of complete information also occurs in the models such as Fernandez and Glazer (2011), Manzini and Mariotti (2004) and Ponsatí and Sákovics (1998). However, the mechanism for delay is different. It arises in those models due to the existence of multiple equilibria. In our model, delay occurs simply because in any period of the bargaining stage, it is a dominant strategy for the responding player to reject the opponent's offer when players are sufficiently patient.[8]

[8] In Yildiz (2011), the arbitration service will never be used in equilibrium. The reason can be explained as follows. In Yilidz (2011), both players know where the arbitrator's ideal settlement is. An implication of this assumption is that for any given Player 1's offer and Player 2's offer, players know which offer will be chosen by the arbitrator if the game moves to the arbitration stage. Suppose it is now in period 2 of the bargaining stage (noting that there are only two periods in the bargaining stage in Yildiz (2011)), in which Player 2 makes a counteroffer. If Player 2's counteroffer is such that Player 1's optimal action is to reject Player 2's counteroffer, then it must be true that Player 1's offer will be chosen by the arbitrator in the arbitration stage (otherwise, if Player 2's counteroffer will be chosen by the arbitrator, then it is meaningless for Player 1 to reject Player 2's counteroffer). This then implies that it is never beneficial for Player 2 to make a counteroffer that will be rejected by Player 1, because Player 2 will be better off by simply making the same offer as Player 1's offer (as such an offer will be accepted by Player 1 and the time cost of going to arbitration is avoided). So, the arbitration service will never be used in Yildiz (2011) (Actually, in Yildiz (2011), not only the arbitration service will never be used in equilibrium, but also players will never reach delayed agreement (i.e., players reach an agreement after period 1 but before the arbitration stage is reached). On the other hand, as shown in Rong (2012), if the arbitrator's ideal settlement is sufficiently favorable to the first mover, then players may reach delayed agreement in equilibrium, although the arbitration stage will also never be reached.)

2.3 Equilibrium payoffs

For any $x_1, x_2 \in [0,1]$ with $x_1 \geq x_2$, define $P(x_2|x_1,x_2) = F(\frac{x_1+x_2}{2})$. That is, $P(x_2|x_1,x_2)$ is the probability that Player 2's offer $(x_2, 1-x_2)$ is chosen as the arbitration outcome when $(x_1, 1-x_1)$ and $(x_2, 1-x_2)$ are the two players' most generous offers. Correspondingly, $1-P(x_2|x_1,x_2)$ is the probability that Player 1's offer $(x_1, 1-x_1)$ is chosen as the arbitration outcome. Let $g_1(x_1,x_2) = U_1(x_1)(1-P(x_2|x_1,x_2)) + U_2(x_2)P(x_2|x_1,x_2)$ and $g_2(x_1,x_2) = U_2(1-x_1)(1-P(x_2|x_1,x_2)) + U_2(1-x_2)P(x_2|x_1,x_2)$ be Player 1's and Player 2's arbitration payoffs, respectively.

We say that the two players' arbitration payoffs are *conflicting* if for any $(x_1,x_2) \in [0,1]\times[0,1]$ and any $(x_1',x_2') \in [0,1]\times[0,1]$, $g_1(x_1',x_2') > g_1(x_1,x_2)$ implies that $g_2(x_1',x_2') < g_2(x_1,x_2)$. Notice that if U_1 and U_2 represent risk-neutral preferences, then the two players' arbitration payoffs are always conflicting, regardless of the distribution of the arbitrator's ideal settlement. Through the remainder of this subsection, we assume that players' utilities and the distribution of the arbitrator's ideal settlement are such that the two players' arbitration payoffs are conflicting.

Let $x_1^* = \underset{x_1\in[0,1]}{argmax}\, g_1(x_1, x_2^*(x_1))$ where $x_2^*(x_1) = \underset{x_2\in[0,x_1]}{argmax}\, g_2(x_1,x_2)$. Imagine that there is a *sequential-offer final-offer arbitration game* in which Player 1 and Player 2 propose offers sequentially (where Player 1 makes the first offer, Player 2 makes the second offer, and players do not need to respond to the opponents' offers), and the arbitrator chooses the offer that is closest to his ideal settlement as the final outcome. In the equilibrium of such a game, Player 1 will make the offer $(x_1^*, 1-x_1^*)$ and Player 2 will make the offer $(x_2^*(x_1^*), 1-x_2^*(x_1^*))$. Let $U_1^* = g_1(x_1^*, x_2^*(x_1^*))$ and $U_2^* = g_2(x_1^*, x_2^*(x_1^*))$. Similarly, we define $x_2^* = \underset{x_2\in[0,1]}{argmax}\, g_2(x_1^*(x_2), x_2)$ where $x_1^*(x_2) = \underset{x_1\in[x_2,1]}{argmax}\, g_1(x_1,x_2)$, and let $U_1^{**} = g_1(x_1^*(x_2^*), x_2^*)$ and $U_2^{**} = g_2(x_1^*(x_2^*), x_2^*)$. It is easy to verify that if the two players' arbitration payoffs are conflicting, then U_1^* and U_2^* are well defined and unique (although x_1^* and $x_2^*(x_1^*)$ may not be unique), and U_1^{**} and U_2^{**} are well defined and unique (although x_2^* and $x_1^*(x_2^*)$ may not be unique).[9] We next

[9]The assumption that the two players' arbitration payoffs are conflicting implies that if $g_1(x_1,x_2) =$

show that in our game, if N is even, then any subgame perfect equilibrium outcome must be such that the two players' (non-discounted) arbitration payoffs are U_1^* and U_2^*, respectively (noting that Theorem 1 guarantees that any subgame perfect equilibrium outcome must be such that the arbitration stage is reached), and if N is odd, then any subgame perfect equilibrium outcome must be such that the two players' (non-discounted) arbitration payoffs are U_1^{**} and U_2^{**}, respectively.

Proposition 1.

(i) If N is even, then the unique subgame perfect equilibrium arbitration payoff is U_1^* for Player 1 and U_2^* for Player 2.

(ii) If N is odd, then the unique subgame perfect equilibrium arbitration payoff is U_1^{**} for Player 1 and U_2^{**} for Player 2.

Although according to Proposition 1, players' equilibrium payoffs are unique, the subgame perfect equilibria of the game may not be unique. Suppose N is even. It can be verified that a subgame perfect equilibrium (outcome) of the game is that Player 1 makes the offer $(x_1^*, 1-x_1^*)$ in any odd period, which Player 2 rejects, and Player 2 makes the offer $(x_2^*(x_1^*), 1-x_2^*(x_1^*))$ in any even period, which Player 1 rejects. Another subgame perfect equilibrium (outcome) is that both players make extreme demands in any period prior to period $N-1$ (which the opponents reject), and then in period $N-1$, Player 1 makes the offer $(x_1^*, 1-x_1^*)$, which Player 2 rejects, and in period N, Player 2 makes the offer $(x_2^*(x_1^*), 1-x_2^*(x_1^*))$, which Player 1 rejects.

Proposition 1 assumes that Player 1 makes the first offer. If Player 2 makes the first offer, then it can be shown that if N is even, the two players' equilibrium (non-discounted) arbitration payoffs are U_1^{**} and U_2^{**}, respectively, and if N is odd, the two players' equilibrium (non-discounted) arbitration payoffs are U_1^* and U_2^* respectively. This result, together with

$g_1(x_1', x_2')$ for some $(x_1, x_2) \in [0,1] \times [0,1]$ and $(x_1', x_2') \in [0,1] \times [0,1]$, then it must be true that $g_2(x_1, x_2) = g_2(x_1', x_2')$. Therefore, although x_1^* and $x_2^*(x_1^*)$ may not be unique, the exact choice of x_1^* and $x_2^*(x_1^*)$ does not matter for players' payoff, which implies that U_1^* and U_2^* are well defined. Similarly, U_1^{**} and U_2^{**} are well defined.

Proposition 1, suggests that a player's equilibrium (non-discounted) arbitration payoff does not depend on whether the player moves first, but whether the player moves last or second-to-last.[10]

2.4 Mover Advantage

We next analyze which player—the last mover or the second-to-last mover—has an advantage? We will not compare the payoffs of Player 1 and Player 2, because the two players' payoffs are largely determined by the degree of bias in the arbitrator's ideal settlement. Instead, we will compare the payoffs of the same player in two scenarios: In one, the player moves second-to-last, and in the other, the player moves last. In particular, let N be fixed, and we will compare Player 1's payoffs across the above two scenarios (i.e., we will compare U_1^* and U_1^{**}).

In general, we may have either $U_1^* \geq U_1^{**}$ or $U_1^* < U_1^{**}$, depending on the exact distribution of the arbitrator's ideal settlement. However, as we will show in the next proposition, for a large class of the distributions of the arbitrator's ideal settlement, we always have $U_1^* \geq U_1^{**}$. That is, the player who moves second-to-last has an advantage in terms of equilibrium arbitration payoff. Let k_1 be the minimum of $\dfrac{U_1'(x)}{U_1(x) - U_1(0)}$ where $x \in [0, 1]$, and k_2 be the minimum of $\dfrac{U_2'(y)}{U_2(y) - U_2(0)}$ where $y \in [0, 1]$, and let $k = \min\{k_1, k_2\}$ (noting that if players are risk-neutral, then $k = 1$).

Proposition 2. *If* $|\dfrac{d \ln f}{dx}| \leq 4k$ *for any* $x \in [0, 1]$*, then we have* $U_1^* \geq U_1^{**}$ *and* $U_2^{**} \geq U_2^*$*. That is, the second-to-last mover has an advantage in arbitration payoff compared to the case in which the player moves last.*

The intuition of Proposition 2 is as follows. Let's first consider an (imaginary)

[10] In particular, the case in which N is even and Player 2 makes the first offer and the case in which N is odd and Player 1 makes the first offer are equivalent in terms of players' (non-discounted) arbitration payoffs (note that in both cases, Player 1 moves last and Player 2 moves second-to-last). Similarly, the case in which N is odd and Player 2 makes the first offer and the case in which N is even and Player 1 makes the first offer are equivalent (note that in both cases, Player 2 moves last and Player 1 moves second-to-last).

simultaneous-offer final-offer arbitration model, in which two players make offers simultaneously and then the arbitrator determines the outcome using final-offer arbitration. One can show that such a model must have one pure-strategy Nash equilibrium if the assumption $|\frac{d \ln f}{dx}| \leq 4k$ is satisfied. For simplicity, let's assume that this pure-strategy Nash equilibrium is also unique and is denoted by $((\hat{x}_1, 1-\hat{x}_1), (\hat{x}_2, 1-\hat{x}_2))$, and let the two players' equilibrium payoffs be $\hat{U}_1$ and $\hat{U}_2$, respectively. Now, let's go back to the sequential-offer final-offer arbitration game. If Player 1 moves second-to-last (i.e., moves first, since there are only two periods in the sequential-offer final-offer arbitration game), Player 1 can always choose to make the offer $(\hat{x}_1, 1-\hat{x}_1)$, in which Player 2 must make the offer $(\hat{x}_2, 1-\hat{x}_2)$ in the last period, because $(\hat{x}_2, 1-\hat{x}_2)$ is a best response to $(\hat{x}_1, 1-\hat{x}_1)$. However, Player 1 may do better by proposing some other offer. So, Player 1's payoff U_1^* must be no less than $\hat{U}_1$. On the other hand, we can show that if Player 1 moves last, then Player 2 can obtain a payoff of at least $\hat{U}_2$, and thus Player 1's payoff U_1^{**} is at most $\hat{U}_1$ (using the fact that the two players' arbitration payoffs are conflicting). Therefore, we must have $U_1^* \geq U_1^{**}$. A similar analysis shows that $U_2^{**} \geq U_2^*$.

The analysis above resembles the analysis of the Stackelberg duopoly model, in which there are two firms, and one firm (the Stackelberg leader) chooses its quantity of output first, and the other firm (the Stackelberg follower) chooses its quantity of output second. The Stackelberg leader usually has an advantage in profit. A striking feature of our model is that only the player who is the Stackelberg leader in the last two periods of bargaining has an advantage, and the order of offers in all previous periods does not matter. Intuitively, this is because the offers made before the last two periods are not credible, in the sense that players can always "revise" their offers in the last two periods.

We next show that if players have the same utility function and the arbitrator is *unbiased* in the sense that the distribution of the arbitrator's ideal settlement is symmetric around 1/2, then it does not matter whether a player moves second-to-last or last.

Corollary 1. *Suppose that* $|\frac{d \ln f}{dx}| \leq 4k$ *for any* $x \in [0,1]$*. If the arbitrator is unbiased (i.e.,* *the distribution of the arbitrator's ideal settlement is symmetric around* $1/2$*) and* $U_1 = U_2$*, then a player obtains the same arbitration payoff no matter whether he moves last or second-to-last.*

2.5 Example

Although Theorem 1 suggests that players' (equilibrium) offers will always be rejected by their opponents, it does not imply that players will make *extreme* offers in equilibrium. Instead, when a player makes an offer, assuming that the player's offer is his most generous offer, there is a trade-off between making a demanding offer (which will bring a larger payoff to the player if the offer is selected by the arbitrator) and making a generous offer (which will make the offer more likely to be selected by the arbitrator).

Next, we consider the situation in which players are risk-neutral and the arbitrator's ideal settlement follows the uniform distribution on $[\frac{1}{2} - \epsilon, \frac{1}{2} + \epsilon]$ with $\epsilon \in (0, \frac{1}{2}]$. For simplicity, we assume that $N = 2$. We will study how the equilibrium offers of the final-offer arbitration game are determined in such a situation. We will also study how a change in the uncertainty of the arbitrator's ideal settlement affects players' equilibrium offers. We have the following result.

Example 1. *Suppose that* $N = 2$*,* $U_1(x_1) = x_1$ *and* $U_2(1 - x_1) = 1 - x_1$*, and* x^* *follows the uniform distribution on* $[\frac{1}{2} - \epsilon, \frac{1}{2} + \epsilon]$ *with* $\epsilon \in (0, \frac{1}{2}]$*. Suppose that* $\delta = 1$*. Then the unique subgame perfect equilibrium of the alternating-offer final-offer arbitration game is such that Player 1 makes the offer* $(\frac{1}{2} + \epsilon, \frac{1}{2} - \epsilon)$*, which Player 2 rejects, and Player 2 makes the counteroffer* $(\frac{1}{2} - \epsilon, \frac{1}{2} + \epsilon)$*, which Player 1 rejects.*

Example 1 demonstrates that when the arbitrator's ideal settlement follows a uniform distribution, the two players' equilibrium offers are the extremes of the uniform distribution (this may not be the case for general distributions). Note that when a player makes a more demanding offer, it has two effects on the player's payoff: a *marginal gain effect*, in the sense that the player will obtain more when the player's offer is chosen by the arbitrator, and a *marginal loss effect*, in the sense that the player's offer will be chosen with a smaller probability. In general, the marginal gain and marginal loss of making a more demanding offer also depend on the other player's offer. The uniform distribution assumption ensures the following two facts. First, the effect of player i's offer on player j's marginal gain of making a more demanding offer exactly offsets the effect of player i's offer on player j's marginal loss of making a more demanding offer. This implies that the optimal decision of player j is irrelevant with respect to player i's offer. Second, for any player, the marginal gain of making a more demanding offer exceeds the corresponding marginal loss if and only if the player's demand is less extreme than the relevant endpoint of the uniform distribution. This implies that the two players will make offers at the extremes of the distribution in equilibrium.

Note that the result in Example 1 is based on the assumption that players are risk-neutral. If players are risk-averse, it is possible for players to make less extreme demands in equilibrium.

Although Stevens (1966) conjectures that uncertainty about the arbitrator's ideal settlement in final-offer arbitration causes players to make more moderate demands during bargaining than they would in conventional arbitration, our example shows that an increase of this uncertainty may cause players' offers to diverge more sharply.[11]

3 Extensions

This section studies several extensions. Its main purpose is to check whether the main result in the basic model—especially the gambling effect of final-offer arbitration—still holds in more realistic settings. We will also study players' bargaining behavior in these new settings.

[11] If players are very risk-averse in the example, then it is possible for players to make more moderate demands when uncertainty about the arbitrator's ideal settlement increases.

3.1 Endogenous ideal settlement

This subsection studies a model in which the arbitrator's ideal settlement is influenced by players' offers. This describes the situation in which the arbitrator may not have full information about the case he arbitrates, or is not fully confident about his ideal settlement, and thus he may rely to some degree on players' offers when making his decision.

More precisely, following the model of Farber and Bazerman (1986) (see also Farber (1981), who considers a special case of Farber and Bazerman (1986)), we assume that the arbitrator's ideal settlement is a weighted average of two components: the arbitrator's private ideal settlement (which is exogenously given) and the weighted average of the offers made by the players. In particular, the arbitrator's ideal settlement, denoted by x^{**}, is determined by the equation $x^{**} = \alpha x^* + (1-\alpha)(\gamma x_1 + (1-\gamma)x_2)$, where x^* is the arbitrator's private ideal settlement, $\alpha \in (0,1)$ measures the arbitrator's confidence in his private ideal settlement x^* (so $1-\alpha$ reflects the influence of players' offers on the arbitrator's ideal settlement), and $\gamma \in (0,1)$ reflects the relative influence of Player 1's offer on the arbitrator's ideal settlement (or the arbitrator's *partiality* toward Player 1).[12] The arbitrator's private ideal settlement, x^*, is only known to the arbitrator, but its distribution is common knowledge among players. We allow α to be dependent on the players' offers. An example is that α is an increasing function of the difference between the two players' proposals (i.e., $x_1 - x_2$). In this case, as the disagreement between players (i.e., the difference between the players' offers) becomes small, the arbitrator relies more on the players' offers to make his decision. For simplicity, we assume that α is known by players.

We call the model described above the *endogenous ideal settlement model*. We obtain the following result, which shows that the main result obtained in the previous section is robust to the introduction of the endogenous ideal settlement.

Proposition 3. *Consider the endogenous ideal settlement model. When players are suffi-*

[12] Note that the partiality of an arbitrator (which is described by γ) is different from the (ex ante) *biasedness* of an arbitrator (which is described by the distribution of x^*).

ciently patient, any subgame perfect equilibrium of the alternating-offer final-offer arbitration game must be such that players will not reach agreement before arbitration, and the arbitration service will be used.

Proposition 3 holds simply because the intuition of the gambling effect of final-offer arbitration discussed in the previous section still holds in the endogenous ideal settlement model. That is, in the endogenous ideal settlement model, it is still true that in any period of the bargaining stage, a player is always better off by rejecting his opponent's offer (and moving the game closer to arbitration) than accepting it, because if the game moves to arbitration, the arbitrator is still restricted to choose between the two players' (most generous) offers. The only change from the basic model is that in final-offer arbitration, for the same combination of players' most generous offers, they are now chosen with a different probability distribution. This change may induce players' equilibrium offers and equilibrium payoffs to be different from the basic model.

Our next result shows that if the arbitrator is *impartial*, in the sense that $\gamma = 1/2$, then players' bargaining behavior and equilibrium payoffs are exactly the same as the basic model.

Corollary 2. *Consider the endogenous ideal settlement model. Suppose that the arbitrator is impartial and players are sufficiently patient. Then the players' equilibrium arbitration payoffs in the alternating-offer final-offer arbitration game are the same as the case in which the arbitrator's ideal settlement is not influenced by the players' offers.*

In final-offer arbitration, a player's offer will be chosen if and only if the player's offer is closer to the arbitrator's ideal settlement than the other player's offer. If the arbitrator is impartial, then the arbitrator's ideal settlement is a weighted average of the arbitrator's private ideal settlement and the *average* (rather than weighted average) of the offers made by the two players. This implies that if a player's offer is closer to the arbitrator's *private ideal settlement* than the other player's offer, then the former player's offer must also be closer to the arbitrator's *ideal settlement* than the latter player's offer. In other words, if the arbitrator is impartial, then the influence of players' offers on the arbitrator's ideal settlement will only change the extent to which an arbitrator favors a player, but is not sufficient to change the arbitrator's choice of favored player. Therefore, players' bargaining behavior and equilibrium payoffs are the same as the situation in which the arbitrator's ideal settlement is not influenced by players' offers.

3.2 Strategic delay

This subsection analyzes the situation in which a player may strategically delay his response to the opponent's offer (we also allow the player who is required to make the first offer to delay his first offer). Clearly, a player's response is an acceptance or a counteroffer (except in the last period, which we will explain shortly). We still assume that the bargaining stage contains N periods. We assume that an offer can be made at the *beginning* of each period, and the response of the offer can be made only at the *end* of this period or the *end* of any later period (note that the end of one period is also the beginning of the next period).[13] At the end of the last period, the responding player can only respond with acceptance or rejection (with no counteroffer).

We assume that Player 1 is required to make the first offer (e.g., in MLB, only the team can make the first offer by offering the player a new contract). If no offer is made during the bargaining stage, then both Player 1 and Player 2 obtain a payoff of zero. If only one offer is made in the bargaining stage and it is rejected or the opponent does not make any response to the offer, then in the arbitration stage the player who did not make an offer needs to submit an offer to the arbitrator, and the arbitrator chooses between the only two offers.

The following result shows that when strategic delay is allowed, players will still not reach agreement before arbitration. In addition, there is a *deadline effect* in bargaining—i.e., players will start to make offers only when the deadline for bargaining is approaching.

[13] The assumption that an offer can be made at the end of (or the beginning of) a period (rather than in the middle of a period) is for simplicity. In the continuous time model we will describe shortly, we allow a player to make an offer at any time, as long as a minimum time has passed since the opponent's last offer.

Proposition 4. *Suppose that* $|\frac{d \ln f}{dx}| \leq 4k$*, players' arbitration payoffs are conflicting, and players are sufficiently patient. Then the subgame perfect equilibrium must be such that the game will move to arbitration. In addition, it is optimal for Player 1 to make his first offer at the beginning of period* $N-1$ *or period* N*.*

According to the analysis in Section 2, a player obtains a higher payoff if he can avoid making the last offer. So, Player 1 has an incentive to delay making his first offer until period $N-1$ or period N, because if Player 1 does so, then Player 2 has to make a counteroffer—either in the last period of the bargaining stage or in the arbitration stage. More importantly, after Player 2 makes the counteroffer, Player 1 cannot respond to Player 2's counteroffer because either the bargaining deadline has been reached (if Player 2 makes the counteroffer in period N) or the game is in the arbitration stage (if Player 2 makes the offer in the arbitration stage). On the other hand, if Player 1 makes an offer in an earlier period, then Player 1 cannot commit not to respond to Player 2's counteroffer, and thus Player 1 will be worse off (more precisely, Player 1 cannot avoid making the last offer, because Player 2 can always choose to make the offer $(x_2^*, 1-x_2^*)$ in period $N-1$ (possibly through delay), and Player 1 has to respond to such an offer by rejecting it and making the counteroffer $(x_1^*(x_2^*), 1-x_1^*(x_2^*)))$.

We can get a result similar to Proposition 4 by considering a continuous time version of the strategic delay model described above. In particular, suppose that the total time of the bargaining stage is T. Player 1 makes the first offer, and Player 1 can make his first offer at any time between 0 and T. We use m to denote the minimum time needed for a player to make a response (i.e., acceptance or a counteroffer). Unlike the discrete model, we assume that after a player makes an offer at time t, the opponent can make a response at *any* time between $t+m$ and T. We assume that at time T (i.e., the end of the bargaining stage), if a player makes a response, then the player can only choose acceptance or rejection (i.e., a counteroffer cannot be made at time T). We also assume that if there is only one offer made in the bargaining stage and the game moves to the arbitration stage, then the player who does not make any offer needs to submit an offer to the arbitrator.

Similar to Proposition 4, one can show that it will be optimal for Player 1 to make his first offer at any time between $T-2m$ and T, so that after Player 2 makes a counteroffer, Player 1 does not have enough time to respond to Player 2's offer. Notice that as m goes to zero, the above interval shrinks to T. We thus have the following result.

Corollary 3. *Suppose that* $|\frac{d\ln f}{dx}| \leq 4k$*, players' arbitration payoffs are conflicting, and players are sufficiently patient. In the continuous-time version of the strategic delay model, as the minimal time needed for a response,* m*, goes to zero, it is optimal for Player 1 to make his first offer at the deadline of the bargaining stage.*

3.3 Strategic submission of offers in the arbitration stage

In reality, when the disputed parties move to final-offer arbitration, players may be given one final chance to make offers. This section studies a variant of the basic model, in which when the game moves to arbitration, each player is allowed to make one more offer (simultaneously with the other player). More precisely, the game studied in this section is as follows.

- **Bargaining stage:** There are N periods in the bargaining stage. Player 1 and Player 2 make alternating offers in the N periods, with Player 1 making the first offer. If, in any period, a player's offer is accepted by the opponent, then the game ends immediately and the accepted offer is the outcome. If the offers in all N periods are rejected, then the game moves to the arbitration stage.

- **Arbitration stage:** Players submit (new) offers simultaneously. If the offers are compatible, then the outcome that splits the difference between the two players' offers is implemented. Otherwise, an arbitrator uses final-offer arbitration to determine the outcome, i.e., the arbitrator chooses either Player 1's submitted offer or Player 2's submitted offer as the outcome.

We assume that *whenever players make new offers, they can only make concessions.* In other words, a player's offer made in any period (including the arbitration stage) should be (weakly) more generous to his opponent than any of the player's previous offers. This reflects the convention of real-life bargaining, in which players usually honor the concessions that they made in the past. We find that the main result of the basic model is unchanged.

Proposition 5. *Consider the alternating-offer final-offer arbitration game in which players can strategically submit new offers in the arbitration stage and players can only make concessions when making new offers. When players are sufficiently patient, in any subgame perfect equilibrium of the game, it must be true that players will not reach agreement before arbitration and the arbitration service will be used.*

Proposition 5 holds for the following reasons. First, note that if the game moves to arbitration, then it is never an equilibrium for players to submit compatible offers. Second, the intuition of the gambling effect of final-offer arbitration still holds. That is, for any offer made by a player (say Player i) in the bargaining stage, acceptance of the offer by Player j will cause the offer to be the outcome, while rejection of the offer will guarantee Player j an outcome that is no worse than the above offer made by Player i. The latter is because Player j can always choose to reject all later offers made by Player i and move the game to arbitration, in which either Player i's offer made in the arbitration stage is chosen or Player j's offer made in the arbitration stage is chosen (noting that Player i's offer made in the arbitration stage must be (weakly) more generous than any previous offer made by Player i in the bargaining stage).

4 Other arbitration formats

4.1 Conventional arbitration

This subsection studies conventional arbitration, in which the arbitrator simply imposes his ideal settlement (i.e., $(x^*, 1 - x^*)$) as the outcome. Similar to the alternating-offer final-offer arbitration game that we define in Section 2.1, we can define the *alternating-offer conventional arbitration game*, in which the arbitrator uses conventional arbitration to determine the outcome if players fail to reach agreement in any of the N periods in the bargaining stage.

For simplicity, let N be an even number. For any given $\delta \in (0, 1]$, let $\hat{x}_2^N(\delta)$ be the unique $x_2 \in [0, 1]$ such that $U_1(x_2) = \delta EU_1(x^*)$, and $\hat{x}_1^{N-1}(\delta)$ the unique $x_1 \in [0, 1]$ such that $U_2(1-x_1) = \delta U_2(1-\hat{x}_2^N(\delta))$. Obviously, $\hat{x}_2^N(\delta)$ gives Player 1 the same (discounted) expected utility payoff as the arbitration outcome. So, if Player 2 makes the offer $(\hat{x}_2^N(\delta), 1 - \hat{x}_2^N(\delta))$ in period N of the bargaining stage, then Player 1 is indifferent between acceptance and rejection. Similarly, according to the definition of $\hat{x}_1^{N-1}(\delta)$, if Player 1 makes the offer $(\hat{x}_1^{N-1}(\delta), 1 - \hat{x}_1^{N-1}(\delta))$ in period $N - 1$ of the bargaining stage, then Player 2 is indifferent between acceptance and rejection. For any $t \in \{1, ..., N - 2\}$ and t is even, we define $\hat{x}_2^t$ as the unique $x_2 \in [0, 1]$ such that $U_1(x_2) = \delta U_1(\hat{x}_1^{t+1})$, and for any $t \in \{1, ..., N - 2\}$ and t is odd, we define $\hat{x}_1^t$ as the unique $x_1 \in [0, 1]$ such that $U_2(1 - x_1) = \delta U_2(1 - \hat{x}_2^{t+1})$. In order to break the tie, we assume that whenever a player is indifferent between acceptance and rejection, the player always chooses acceptance. We also assume that whenever a player is indifferent between making an offer that the other player accepts and making an offer that the other player rejects, he will make the former offer. We have the following result.

Proposition 6. *In the alternating-offer conventional arbitration game, for any given* $\delta \in (0, 1]$*, the unique subgame perfect equilibrium is such that Player 1 makes the offer* $(\hat{x}_1^1(\delta), 1 - \hat{x}_1^1(\delta))$ *in period 1, which Player 2 accepts.*

The proof of Proposition 6 is simply a result of backward induction (a detailed proof is in the Appendix). Note that if we assume that the arbitrator's ideal settlement is known to the players, then we can obtain a similar result in which players reach agreement in period 1 of the bargaining stage. This implies that when the arbitration mechanism is conventional arbitration, whether the arbitrator's ideal settlement is known to players or not does not matter too much for players' bargaining behavior. In conventional arbitration, the arbitration outcome is exogenously given and what players care about are just the expected payoffs they obtain from arbitration.

4.2 Modified final-offer arbitration

In our basic model, a key assumption is that the arbitrator will only consider either Player 1's *most generous* offer or Player 2's *most generous* offer. In this subsection, we drop this assumption and consider the final-offer arbitration in which the arbitrator simply chooses the offer that is closest to the arbitrator's ideal settlement (among all N offers made by players in the bargaining stage). Note that the offer that is closest to the arbitrator's ideal settlement may not be the most generous offer of any player. With this modified final-offer arbitration, the gambling effect of final-offer arbitration will still hold for the case where there are two periods in the bargaining stage (i.e., $N = 2$), because in this case, each player can make only one offer in the bargaining stage, which must also be the player's most generous offer. However, for the general case where $N > 2$, the gambling effect may not hold and players may reach agreement before arbitration. We will illustrate this point using an example (where the proof is in the Appendix).

Example 2. *Suppose that* $N = 3$, $U_1(x_1) = x_1$ *and* $U_2(1 - x_1) = 1 - x_1$, *and* x^* *follows the uniform distribution on* $[0, 1]$. *Suppose that* $\delta = 1$. *Then a subgame perfect equilibrium of the alternating-offer with modified final-offer arbitration game is such that Player 1 makes the offer* $(1, 0)$ *in period 1, which Player 2 rejects; Player 2 makes the offer* $(0, 1)$ *in period 2, which Player 1 rejects; and Player 1 makes the offer* $(1/2, 1/2)$ *in period 3, which Player 2 accepts.*

5 Conclusion

In this paper, we study a finite-horizon alternating-offer bargaining model in which if players fail to reach agreement in a finite period of time, then the game moves to final-offer arbitration (in which the arbitrator chooses either Player 1's most generous offer or Player 2's most generous offer). We find that in any equilibrium of the game, players will never reach agreement before arbitration, and thus the arbitration service is always used.[14] This is in contrast to the popular belief that the threat of going to final-offer arbitration encourages players to reach agreement before arbitration. Our result also holds in various extensions, including allowing endogenous ideal settlement, allowing strategic delay in bargaining, and allowing strategic submission of new offers in arbitration.

It is worthy to note that our result relies on some key assumptions. First, there are no arbitration fees in our model. Second, the players are assumed to be sufficiently patient, i.e., the time cost of delay is small. If any of these assumptions is violated, our result will not hold. In this sense, our model is not intended to overturn the use of final-offer arbitration in reality, but rather emphasizes how the absence of high arbitration costs may undermine the effectiveness of final-offer arbitration in practice.

Appendix: Proofs

Proof of Lemma 1:

Suppose there exists some $1 \leq t \leq N$ such that $x_1^{(t)} \leq x_2^{(t)}$. Let $t_1 = \min\{1 \leq s \leq t : x_1^s = x_1^{(t)}\}$ and $t_2 = \max\{1 \leq s \leq t : x_2^s = x_2^{(t)}\}$. Assume that $t_1 < t_2$ (the proof of the other case (i.e., $t_1 > t_2$) is similar). That is, Player 1 makes the offer $(x_1^{t_1}, 1 - x_1^{t_1}) = (x_1^{(t)}, 1 - x_1^{(t)})$ in period t_1, which Player 2 rejects, and Player 2 makes the offer $(x_2^{t_2}, 1 - x_2^{t_2}) = (x_2^{(t)}, 1 - x_2^{(t)})$ in some later period t_2, where $1 - x_1^{t_1} \geq 1 - x_2^{t_2}$. This cannot be an equilibrium outcome,

[14] When final-offer arbitration is the arbitration mechanism, we usually prefer that players reach agreement by themselves rather than resort to arbitration, because (i) the arbitration process is time-costly, and (ii) if the game moves to final-offer arbitration, then the outcome is usually (ex post) “unfair” because a compromised outcome is not allowed.

because (i) if Player 2 chooses to accept Player 1's offer $(x_1^{t_1}, 1-x_1^{t_1})$ in period t_1, then Player 2's payoff is $\delta^{t_1-1}U_2(1-x_1^{t_1})$, and (ii) if Player 2 rejects Player 1's offer $(x_1^{t_1}, 1-x_1^{t_1})$ in period t_1 and makes an offer $(x_2^{t_2}, 1-x_2^{t_2})$ in a later period t_2, then Player 2's payoff must not be greater than $\delta^{t_2-1}U_2(1-x_2^{t_2})$(because Player 1 can guarantee a payoff of $U_1(x_2^{t_2})$ in period t_2 by accepting the offer $(x_2^{t_2}, 1-x_2^{t_2})$). Since $\delta^{t_1-1}U_2(1-x_1^{t_1}) \geq \delta^{t_2-1}U_2(1-x_2^{t_2})$, Player 2 should accept Player 1's offer in period t_1.[15] □

Proof of Theorem 1:

We consider the simple case in which both players are perfectly patient, i.e., the discount factor $\delta = 1$. The proof for the more general (but much more complicated) case where δ is less than but is sufficiently close to 1 is put at the end of the Appendix.

Suppose N is even (the proof for the case where N is odd is similar). To gain some intuition of the analysis, suppose that the game now moves to period N. The most generous offer that Player 1 has ever made up to period $N-1$ is $(x_1^{(N-1)}, 1-x_1^{(N-1)})$, and the most generous offer that Player 2 has ever made up to period $N-1$ is $(x_2^{(N-1)}, x_2^{(N-1)})$. Then we must have $x_2^{(N-1)} < x_1^{(N-1)}$ (see Lemma 1). Let the offer made by Player 2 in period N be $(x_2^N, 1-x_2^N)$. If $x_2^N \geq x_1^{(N-1)}$, then $x_2^{(N)} = \max\{x_2^{(N-1)}, x_2^N\} \geq x_1^{(N-1)} = x_1^{(N)}$. This is a contradiction with Lemma 1. So, we must have $x_2^N < x_1^{(N-1)}$. For Player 1, accepting the offer $(x_2^N, 1-x_2^N)$ will simply result in $(x_2^N, 1-x_2^N)$ as the outcome, while rejecting the offer will move the game to arbitration, in which either $(x_1^{(N)}, 1-x_1^{(N)}) = (x_1^{(N-1)}, 1-x_1^{(N-1)})$ or $(x_2^{(N)}, 1-x_2^{(N)})$ will be chosen as the arbitration outcome. So, Player 1 is better off by rejecting Player 2's offer because $x_1^{(N-1)} > x_2^N$ and $x_2^{(N)} = \max\{x_2^{(N-1)}, x_2^N\} \geq x_2^N$.

The above reasoning applies to any previous periods. Suppose the game is now in some period $t < N$. Suppose t is odd (the case that t is even is similar). Notice that $(x_1^{(t-1)}, 1-x_1^{(t-1)})$ is the most generous offer of Player 1 up to period $t-1$, and $(x_2^{(t-1)}, 1-x_2^{(t-1)})$ is the

[15] If $\delta^{t_1-1}U_2(1-x_1^{t_1}) = \delta^{t_2-1}U_2(1-x_2^{t_2})$ (which occurs when $(1-x_1^{t_1}) = 1-x_2^{t_2}$ and $\delta = 1$), then Player 1 is indifferent between accepting Player 1's offer in period t_1 and rejecting it. In this case, Player 1 chooses acceptance (by the tie-breaking rule).

most generous offer of Player 2 up to period $t-1$.[16] Then, we must have $x_2^{(t-1)} < x_1^{(t-1)}$ by Lemma 1. Let the offer made by Player 1 in period t be $(x_1^t, 1-x_1^t)$. If $x_1^t \leq x_2^{(t-1)}$ (i.e., $1-x_1^t \geq 1-x_2^{(t-1)}$), then $x_1^{(t)} = \min\{x_1^{(t-1)}, x_1^t\} \leq x_2^{(t-1)} = x_2^{(t)}$, which violates Lemma 1. So, we must have $x_1^t > x_2^{(t-1)}$. It is optimal for Player 2 to reject such an offer. This is because the acceptance of the offer will simply result in $(x_1^t, 1-x_1^t)$ as the outcome, while rejecting the offer will bring Player 2 a payoff that is strictly greater than $1-x_1^t$, because Player 2 can reject the offer and then makes the offer $(x_2^{(t-1)}, 1-x_2^{(t-1)})$ in all later periods (either Player 1 accepts the offer in some later period or the game moves to arbitration, in which Player 2's most generous offer $(x_2^{(N)}, 1-x_2^{(N)})$ (where $x_2^{(N)} = x_2^{(t-1)} < x_1^t$) or Player 1's most generous offer $(x_1^{(N)}, 1-x_1^{(N)})$ (where $x_1^{(N)} \leq x_1^t$) will be chosen; in either case, Player 2's payoff is greater than $1-x_1^t$). So, Player 2 is better off rejecting Player 1's offer.

So, we have shown that if $\delta = 1$, then in equilibrium, players will make an offer that will be rejected by the opponents in all N periods. □

Proof of Proposition 1:

Suppose N is even. Player 1 can guarantee a payoff of $\delta^N U_1^*$ because Player 1 can propose x_1^* in any odd period t for $1 \leq t \leq N$ (regardless of Player 2's strategy), and (i) if the game moves to arbitration, Player 1's most generous offer will be $(x_1^*, 1-x_1^*)$, and Player 1's (non-discounted) arbitration payoff must be no less than U_1^*, and (ii) if the game ends before the arbitration stage, then it must be the case that Player 1 obtains a payoff higher than $\delta^N U_1^*$.

Player 2 can guarantee a payoff of $\delta^N U_2^*$ because Player 2 can propose $(0,1)$ in any even period t for $1 \leq t \leq N-2$. To see this point, notice that if Player 2 proposes $(0,1)$ in all even periods t for $1 \leq t \leq N-2$, then Player 1's optimal offer in period $N-1$ must be x_1^*, which Player 2 rejects and proposes $(x_2^*(x_1^*), 1-x_2^*(x_1^*))$ in period N.

So, we have shown that in any subgame perfect equilibrium outcome, Player 1's payoff should be no less than $\delta^N U_1^*$ and Player 2's payoff should be no less than $\delta^N U_2^*$. According to Theorem 1, all equilibria must involve arbitration. Since the two players' arbitration payoffs

[16] If $t=1$, then let $x_2^{(t-1)} = 0$ and $x_1^{(t-1)} = 1$.

are conflicting, the subgame perfect equilibrium (non-discounted) arbitration payoff must be unique and must be U_1^* for Player 1 and U_2^* for Player 2.

The proof for the case where N is odd is similar and is omitted. $\square$

Proof of Proposition 2:

Consider a *simultaneous-offer final-offer arbitration game*, in which two players make offers simultaneously and the arbitrator determines the outcome using final-offer arbitration. We now solve the Nash equilibrium of such a game. Let $\hat{x}_1(x_2)$ be Player 1's best response function and $\hat{x}_2(x_1)$ Player 2's best response function. Then we must have:[17]

$$\hat{x}_1(x_2) = argmax_{x_1 \in [x_2,1]} U_1(x_1)(1 - F(\frac{x_1 + x_2}{2})) + U_1(x_2)F(\frac{x_1 + x_2}{2}) \tag{1}$$

and

$$\hat{x}_2(x_1) = argmax_{x_2 \in [0,x_1]} U_2(1 - x_1)(1 - F(\frac{x_1 + x_2}{2})) + U_2(1 - x_2)F(\frac{x_1 + x_2}{2}). \tag{2}$$

The first-order condition of Equation 1 is $U_1'(x_1)(1 - F(\frac{x_1 + x_2}{2})) - (U_1(x_1) - U_1(x_2))f(\frac{x_1 + x_2}{2})\frac{1}{2} = 0$ and the second-order condition is $U_1''(x_1)(1 - F(\frac{x_1 + x_2}{2})) - U_1'(x_1)f(\frac{x_1 + x_2}{2}) - \frac{1}{4}(U_1(x_1) - U_1(x_2))f'(\frac{x_1 + x_2}{2}) \leq 0$. The second-order condition is satisfied under the assumption that $\frac{d \ln f}{dx} \geq -4k$ (i.e., $\frac{f'}{f} \geq -4k$) (and using the fact that $U_1''(x_1) \leq 0$ and $U_1(x_2) \geq U_1(0)$).

The first-order condition of Equation 2 is $-U_2'(1 - x_2)F(\frac{x_1 + x_2}{2}) + (U_2(1 - x_2) - U_2(1 - x_1))f(\frac{x_1 + x_2}{2})\frac{1}{2} = 0$ and the second-order condition is $U_2''(1 - x_2)F(\frac{x_1 + x_2}{2}) - U_2'(1 - x_2)f(\frac{x_1 + x_2}{2}) + \frac{1}{4}(U_2(1 - x_2) - U_2(1 - x_1))f'(\frac{x_1 + x_2}{2}) \leq 0$. This second-order condition is also satisfied under the assumption that $\frac{d \ln f}{dx} \leq 4k$ (i.e., $\frac{f'}{f} \leq 4k$) (and using the fact that $U_2''(1 - x_2) \leq 0$ and $U_2(1 - x_1) \geq U_2(0)$).

[17]Note that it is never a best response for Player 1 to propose an offer such that $x_1 < x_2$. Similarly, it is never optimal for Player 2 to propose an offer such that $x_2 > x_1$. So, we put the restriction $x_1 \geq x_2$ in the following maximization problems.

We thus have shown that the first-order conditions of both players are also sufficient. Note that the maximization problems in Equations 1 and 2 have solutions because they are maximization problems of continuous functions in compact sets. So, both the best response function of Player 1 and the best response function of Player 2 are well defined. In addition, since f and U_i' are differentiable, the best response functions of both players are continuous. Using the Brouwer fixed point theorem, it can be shown that the pure-strategy Nash equilibrium exists.

We use $((\hat{x}_1, 1-\hat{x}_1), (\hat{x}_2, 1-\hat{x}_2))$ to denote the (pure-strategy) Nash equilibrium that brings Player 1 the highest payoff among all (pure-strategy) Nash equilibria, and we denote the two players' payoffs in this equilibrium $\hat{U}_1$ and $\hat{U}_2$, respectively. Similarly, we use $((\hat{x}_1', 1-\hat{x}_1'), (\hat{x}_2', 1-\hat{x}_2'))$ to denote the (pure-strategy) Nash equilibrium that brings Player 2 the highest payoff among all (pure-strategy) Nash equilibria, and we denote the two players' payoffs in this equilibrium $\hat{U}_1'$ and $\hat{U}_2'$, respectively. Then, we must have $\hat{U}_1' \leq \hat{U}_1$. Now, let's consider the *sequential-offer final-offer arbitration game*, in which the two players make offers sequentially (and do not need to choose acceptance or rejection), and the arbitrator uses final-offer arbitration to determine the outcome. Suppose Player 1 makes the first offer. Then Player 1's equilibrium payoff U_1^* must be at least $\hat{U}_1$ (because Player 1 can at least make the offer $(\hat{x}_1, 1-\hat{x}_1)$). That is, $U_1^* \geq \hat{U}_1$. Now, suppose Player 1 makes the second offer, then Player 2's equilibrium payoff U_2^{**} must be at least $\hat{U}_2'$, which implies that Player 1's equilibrium payoff U_1^{**} is at most $\hat{U}_1'$ (because the two players' arbitration payoffs are conflicting). Since $\hat{U}_1'$ is not greater than $\hat{U}_1$, we have $U_1^{**} \leq \hat{U}_1$. Therefore, we have shown that $U_1^* \geq U_1^{**}$.

Similarly, we can show that $U_2^{**} \geq U_2^*$. □

Proof of Corollary 1:

For simplicity, we assume that players have risk-neutral preferences. In particular, we assume that $U_1(x) = U_2(x) = x$, in which the sum of the two players' arbitration payoffs is always 1.

Suppose Player 2 moves last. For any Player 1's most generous offer up to period $N-1$ (i.e, $(x_1^{(N-1)}, 1-x_1^{(N-1)})$), Player 2 can always propose a "symmetric" offer $(x_2^N, 1-x_2^N) = (1-x_1^{(N-1)}, x_1^{(N-1)})$. Such an offer will guarantee Player 2 an arbitration payoff of 1/2 in the arbitration stage. So, Player 2's equilibrium arbitration payoff must be no less than 1/2. But Player 2's equilibrium arbitration payoff cannot be strictly greater than 1/2. This is because if it is strictly greater than 1/2, then Player 1's equilibrium arbitration payoff will be strictly less than 1/2. However, if Player 1 moves last, then Player 1's payoff will also be strictly greater than 1/2 (due to the symmetry of the problem), and this contradicts the fact that Player 1, as the second-to-last mover, has an advantage in arbitration payoff. So, Player 2's arbitration payoff must be 1/2, and thus Player 1's arbitration payoff is also 1/2. Similarly, if Player 2 moves second-to-last, then both players' arbitration payoffs are also 1/2.

The analysis above can be easily extended to the general case where the two players' utility functions are the same. □

Proof of Example 1:

Since there are only two periods in the bargaining stage, we simply use $(x_1, 1-x_1)$ to denote Player 1's offer (which is made in period 1), and $(x_2, 1-x_2)$ to denote Player 2's offer (which is made in period 2).

Suppose it is in period 2. Note that the offer that Player 1 makes in period 1 must be such that $x_1 > \frac{1}{2} - \epsilon$ (otherwise, if $x_1 \leq \frac{1}{2} - \epsilon$, then Player 2 must accept the offer in equilibrium because the ideal point of the arbitrator can never be less than $\frac{1}{2} - \epsilon$). In period 2, it is never optimal for Player 2 to make an offer $(x_2, 1-x_2)$ with either $\frac{x_1+x_2}{2} < \frac{1}{2} - \epsilon$ or $\frac{x_1+x_2}{2} > \frac{1}{2} + \epsilon$.[18] Thus, $(x_2, 1-x_2)$ must be such that $\frac{1}{2} - \epsilon \leq \frac{x_1+x_2}{2} \leq \frac{1}{2} + \epsilon$. Also, notice that it is never optimal for Player 2 to make an offer $(x_2, 1-x_2)$ with $x_2 \geq x_1$. We

[18] In the former case, Player 2 will obtain a higher payoff by making the offer $(x_2, 1-x_2)$ with $x_2 = \frac{1}{2} - \epsilon$. In the latter case, if $x_2 \leq x_1$, Player 2 will obtain a higher payoff by making a slightly more demanding offer; if $x_2 > x_1$, Player 2 will obtain a higher payoff by making the offer $(x_2, 1-x_2)$ with $x_2 = \frac{1}{2} - \epsilon$.

thus have $x_2 < x_1$. The fact that $\frac{1}{2} - \epsilon \leq \frac{x_1 + x_2}{2} \leq \frac{1}{2} + \epsilon$ and $x_2 < x_1$ implies that Player 1 will reject $(x_2, 1 - x_2)$ in equilibrium. This is because rejection of Player 2's offer will move the game to the arbitration stage, in which either Player 1's offer $(x_1, 1 - x_1)$ or Player 2's offer $(x_2, 1 - x_2)$ is the outcome, while the acceptance of Player 2's offer will simply result in $(x_2, 1 - x_2)$ as the outcome.

We next calculate the optimal offer of Player 2. Since $\frac{1}{2} - \epsilon \leq \frac{x_1 + x_2}{2} \leq \frac{1}{2} + \epsilon$, we have $P(x_2|x_1, x_2) = F(\frac{x_1 + x_2}{2}) = \frac{1}{2\epsilon}[\frac{x_1 + x_2}{2} - (\frac{1}{2} - \epsilon)]$. Let $(x_2^*(x_1), 1 - x_2^*(x_1))$ be the optimal offer of Player 2. Then we must have:

$$
\begin{aligned}
x_2^*(x_1) &= \underset{x_2 \in [0, x_1)}{argmax}\{U_2(1 - x_1)(1 - P(x_2|x_1, x_2)) + U_2(1 - x_2)P(x_2|x_1, x_2)\} \\
&= \underset{x_2 \in [0, x_1)}{argmax}\{(1 - x_1)(1 - P(x_2|x_1, x_2)) + (1 - x_2)P(x_2|x_1, x_2)\} \\
&= \underset{x_2 \in [0, x_1)}{argmax}\{1 - x_1 + P(x_2|x_1, x_2)(x_1 - x_2)\} \\
&= \underset{x_2 \in [0, x_1)}{argmax}\{1 - x_1 + \frac{1}{2\epsilon}[\frac{x_1 + x_2}{2} - (\frac{1}{2} - \epsilon)](x_1 - x_2)\}.
\end{aligned}
$$

It can be verified that

$$
x_2^*(x_1) = \begin{cases} \frac{1}{2} - \epsilon & if \quad \frac{1}{2} - \epsilon < x_1 < \frac{1}{2} + 3\epsilon; \\ 1 + 2\epsilon - x_1 & if \quad x_1 \geq \frac{1}{2} + 3\epsilon. \end{cases}
$$

Now, let's go back to period 1. We have the following three cases.

(i) If Player 1's offer $(x_1, 1 - x_1)$ is such that $x_1 \geq \frac{1}{2} + 3\epsilon$, then Player 2 will reject the offer. This is because if Player 2 chooses to accept, then Player 2's payoff is $1 - x_1$. If, instead, Player 2 chooses to reject, then Player 2 will offer $x_2^*(x_1) = 1 + 2\epsilon - x_1$ in period 2, which Player 1 rejects in equilibrium, and Player 2's payoff is $1 - x_1 + \frac{1}{2\epsilon}[\frac{x_1 + x_2^*(x_1)}{2} - (\frac{1}{2} - \epsilon)](x_1 - x_2^*(x_1)) = x_1 - 2\epsilon$, which is strictly greater than $1 - x_1$. Player 1's equilibrium payoff is thus $1 - (x_1 - 2\epsilon) = 1 - x_1 + 2\epsilon$. Notice that Player 1's payoff in this case is not greater than $1 - (\frac{1}{2} + 3\epsilon) + 2\epsilon = \frac{1}{2} - \epsilon$.

(ii) If Player 1's offer $(x_1, 1 - x_1)$ is such that $\frac{1}{2} - \epsilon < x_1 < \frac{1}{2} + 3\epsilon$, then Player 2 will reject the offer. This is because if Player 2 chooses to accept, then his payoff is $1 - x_1$. If, instead, Player 2 chooses to reject, then Player 2 will offer $x_2^*(x_1) = \frac{1}{2} - \epsilon$ in period 2, which Player 1 rejects in equilibrium, and Player 2's payoff is $1 - x_1 + \frac{1}{2\epsilon}[\frac{x_1 + x_2^*(x_1)}{2} - (\frac{1}{2} - \epsilon)](x_1 - x_2^*(x_1)) = 1 - x_1 + \frac{1}{4\epsilon}(x_1 - \frac{1}{2} + \epsilon)^2$, which is strictly greater than $1 - x_1$. Player 1's payoff is thus $x_1 - \frac{1}{4\epsilon}(x_1 - \frac{1}{2} + \epsilon)^2$. In this case, Player 1's payoff reaches its maximum $\frac{1}{2}$ when $x_1 = \frac{1}{2} + \epsilon$.

(iii) If Player 1's payoff $(x_1, 1 - x_1)$ is such that $x_1 \leq \frac{1}{2} - \epsilon$, Player 2 will accept the offer. Player 1's payoff is thus x_1. In this case, Player 1's maximum payoff is $\frac{1}{2} - \epsilon$.

Based on the analysis above, we can see that the unique equilibrium is such that Player 1 makes the offer $(x_1, 1 - x_1)$ with $x_1 = \frac{1}{2} + \epsilon$, which Player 2 rejects, and Player 2 makes the counteroffer $(x_2, 1 - x_2)$ with $x_2 = \frac{1}{2} - \epsilon$, which Player 1 rejects. □

Proof of Corollary 2:

Let $P(x_2|x_1, x_2, x^*)$ be the probability that Player 2's offer is chosen as the final-offer arbitration outcome when Player 1's offer is (x_1, y_1) and Player 2's offer is (x_2, y_2) and the arbitrator's ideal settlement does not have an average offer component (i.e., the arbitrator's ideal settlement is x^*). On the other hand, let $P(x_2|x_1, x_2, x^{**})$ be the probability that Player 2's offer is chosen as the final-offer arbitration outcome when Player 1's offer is (x_1, y_1), Player 2's offer is (x_2, y_2), and the arbitrator's ideal settlement has an average offer component (i.e., the arbitrator's ideal settlement is x^{**}). Then, we have $P(x_1|x_1, x_2, x^{**}) = Prob(x^{**} < \frac{x_1 + x_2}{2}) = Prob(\alpha x^* + (1 - \alpha)\frac{x_1 + x_2}{2} < \frac{x_1 + x_2}{2}) = Prob(x^* < \frac{x_1 + x_2}{2}) = P(x_2|x_1, x_2, x^*)$. That is, the probability that Player 2's offer is chosen when the arbitrator's ideal settlement is x^{**} is exactly the same as the probability that Player 2's offer is chosen when the arbitrator's ideal settlement is x^*. This implies that the equilibrium outcome of the alternating-offer final-offer arbitration game when the arbitrator's ideal settlement is x^{**} should be the same as the final-offer arbitration game when the arbitrator's ideal settlement is x^*. The remaining proof is the same with the proof of Theorem 1 and is omitted. □

Sketch of Proof of Proposition 4:

If Player 1 makes the first offer at the beginning of period $N - 1$, then Player 1 can guarantee an (arbitration) payoff of U_1^* because Player 1 can make the offer $(x_1^*, 1 - x_1^*)$ at the beginning of period $N-1$ (and Player 2 must choose to reject and make the counteroffer $(x_2^*(x_1^*), 1 - x_2^*(x_1^*))$). It can be verified that Player 1 cannot get a higher payoff. This is because if Player 1 makes an offer in a period earlier than period $N - 1$, then Player 2 can always choose to reject it and make the offer $(x_2^*, 1 - x_2^*)$ at the beginning of period $N - 1$ and thus guarantees a payoff of U_2^{**}, which implies that Player 1's payoff in this case must be not greater than U_1^{**}, which is less than U_1^*.

Similarly, we can show that it is optimal for Player 1 to make the offer at the beginning of period N (and Player 2 will reject such an offer, and in the arbitration stage, Player 2 will submit the offer $(x_2^*(x_1^*), 1 - x_2^*(x_1^*))$). $\square$

Proof of Proposition 6:

For simplicity, we focus on the case where $N = 2$ (and the analysis below can be easily extended to the general case where $N > 2$). In order to simplify the notation, we write $\hat{x}_2^2(\delta)$ as $\hat{x}_2(\delta)$ and $\hat{x}_1^1(\delta)$ as $\hat{x}_1(\delta)$.

Suppose it is in period 2. Let $(x_2, 1 - x_2)$ be the offer made by Player 2. Obviously, if $x_2 \geq \hat{x}_2(\delta)$, then Player 1 accepts the offer $(x_2, 1-x_2)$ in equilibrium, and Player 2's maximum possible payoff in this case is $U_2(1 - \hat{x}_2(\delta))$. If $x_2 < \hat{x}_2(\delta)$, then Player 1 rejects the offer $(x_1, 1-x_1)$ in equilibrium, and the game moves to the arbitration stage and Player 2's payoff is $\delta EU_2(1-x^*)$. Since $U_1(\hat{x}_2(\delta)) = \delta EU_1(x^*) \leq \delta U_1(Ex^*)$ (where the equality is by definition and the inequality follows from the fact that U_1 is concave), we must have $\hat{x}_2(\delta) \leq Ex^*$. We thus have $U_2(1 - \hat{x}_2(\delta)) \geq U_2(1 - Ex^*) = U_2(E(1 - x^*)) \geq EU_2(1 - x^*) \geq \delta EU_2(1 - x^*)$, where the second to the last inequality follows from the fact that U_2 is concave. Thus, in period 2, the optimal offer of Player 2 is $(\hat{x}_2(\delta), 1 - \hat{x}_2(\delta))$, which Player 1 chooses to accept in equilibrium.

Now, let's go back to period 1. Let $(x_1, 1 - x_1)$ be the offer made by Player 1. Obviously, if $x_1 \leq \hat{x}_1(\delta)$, then Player 2 chooses to accept $(x_1, 1 - x_1)$ in equilibrium, and Player 1's maximum possible payoff in this case is $U_1(\hat{x}_1(\delta))$. If $x_1 > \hat{x}_1(\delta)$, then Player 2 chooses to reject $(x_1, 1 - x_1)$ in equilibrium, and Player 1's payoff in this case is $\delta U_1(\hat{x}_2(\delta))$. By definition, we have $U_2(1 - \hat{x}_1(\delta)) = \delta U_2(1 - \hat{x}_2(\delta))$, which implies that $\hat{x}_1(\delta) \geq \hat{x}_2(\delta)$. Thus, we have $U_1(\hat{x}_1(\delta)) \geq \delta U_1(\hat{x}_2(\delta))$, and Player 1's optimal offer in period 1 is $(\hat{x}_1(\delta), 1 - \hat{x}_1(\delta))$, which Player 2 chooses to accept in equilibrium. $\square$

Proof of Example 2:

Suppose it is in period 3. Let $(x_1^1, 1 - x_1^1)$ be Player 1's offer made in period 1, and $(x_2^2, 1 - x_2^2)$ be Player 2's offer made in period 2. Notice that it is never optimal for Player 2 to make an offer $(x_2^2, 1 - x_2^2)$ with $x_2^2 \geq x_1^1$. We thus have $x_2^2 < x_1^1$. Let Player 1's offer in period 3 be $(x_1^3, 1 - x_1^3)$. If Player 2 accepts Player 1's offer $(x_1^3, 1 - x_1^3)$, then Player 2's payoff will be $1 - x_1^3 := U_2^A(x_1^3)$. If Player 2 rejects Player 1's offer $(x_1^3, 1 - x_1^3)$, then the game will move to arbitration and Player 2's payoff will be $(1 - x_1^1)P(x_1^1|x_1^1, x_2^2, x_1^3) + (1 - x_2^2)P(x_2^2|x_1^1, x_2^2, x_1^3) + (1 - x_1^3)P(x_1^3|x_1^1, x_2^2, x_1^3) := U_2^R(x_1^1, x_2^2, x_1^3)$, where $P(x_i^t|x_1^1, x_2^2, x_1^3)$ is the probability that Player i's offer in period t is chosen as the arbitration outcome. Note that x_1^3 must be such that $x_2^2 \leq x_1^3 \leq x_1^1$,[19] which implies that $P(x_1^1|x_1^1, x_2^2, x_1^3) = 1 - F(\dfrac{x_1^3 + x_1^1}{2}) = 1 - \dfrac{x_1^3 + x_1^1}{2}$, $P(x_2^2|x_1^1, x_2^2, x_1^3) = F(\dfrac{x_1^3 + x_2^2}{2}) = \dfrac{x_1^3 + x_2^2}{2}$, and $P(x_1^3|x_1^1, x_2^2, x_1^3) = F(\dfrac{x_1^3 + x_1^1}{2}) - F(\dfrac{x_1^3 + x_2^2}{2}) = \dfrac{x_1^1 - x_2^2}{2}$ (see also Figure 1). It can be verified that $U_2^R(x_1^1, x_2^2, x_1^3) = 1 - x_1^1 + \dfrac{(x_1^1)^2}{2} - \dfrac{(x_2^2)^2}{2}$. Interestingly, U_2^R is independent of x_1^3. In addition, U_2^A is decreasing in x_1^3. So, there must exist a unique x_1^3, denoted by $x_1^{3*}(x_1^1, x_2^2)$ (or simply, x_1^{3*}, when there is no confusion), such that if $x_1^3 \leq x_1^{3*}$, then Player 2 will accept $(x_1^3, 1 - x_1^3)$, and if $x_1^3 > x_1^{3*}$, then Player 2 will reject $(x_1^3, 1 - x_1^3)$ (see also the left panel of Figure 2).[20] Thus, for Player 1, $(x_1^{3*}, 1 - x_1^{3*})$ will be his optimal offer in period 3 (see also the right panel of Figure 2).[21]

We now go back to period 2. Suppose Player 2 makes the offer $(x_2^2, 1 - x_2^2)$ in period 2 (with $x_2^2 < x_1^1$), then it is optimal for Player 1 to reject the offer and the game will move to

[19] If $x_1^3 > x_1^1$, then Player 2 must reject the offer $(x_1^3, 1 - x_1^3)$, and thus Player 1 will be better off by making the offer with $x_1^3 = x_1^1$ (which Player 2 also rejects). If $x_1^3 < x_2^2$, then Player 2 must accept the offer $(x_1^3, 1 - x_1^3)$, and Player 1 is still better off by making the offer with $x_1^3 = x_1^1$ (which Player 2 rejects).

[20] The existence of $x_1^{3*}(x_1^1, x_2^2)$ is due to the following two observations: (i) if $x_1^3 = x_2^2$, then Player 2 must accept the offer, and (ii) if $x_1^3 = x_1^1$, then Player 2 must reject the offer.

[21] The optimal offer is not unique. More precisely, any offer $(x_1^3, 1 - x_1^3)$ with $x_1^{3*} \leq x_1^3 \leq x_1^1$ is optimal.

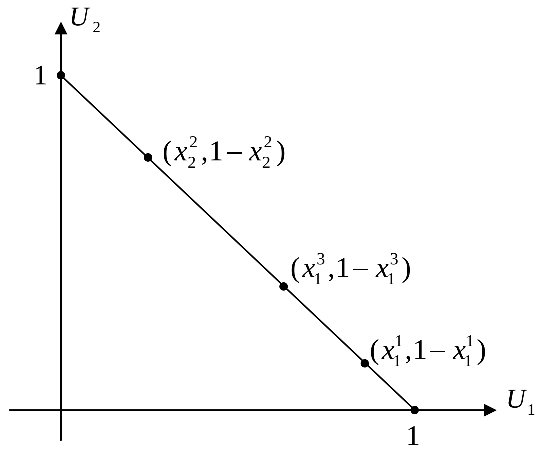

Figure 1: An illustration of players' offers.

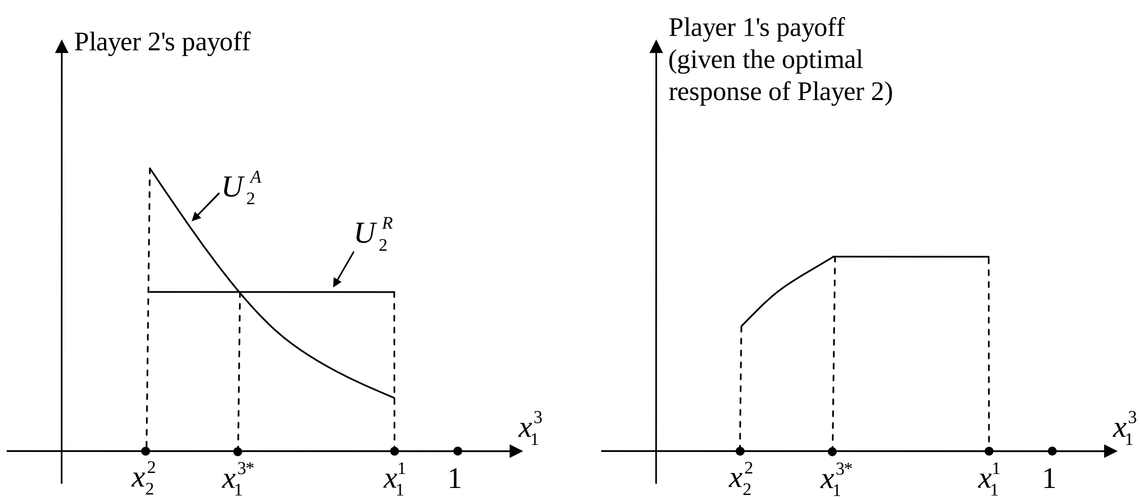

Figure 2: Players' payoffs as a function of x_1^3.

period 3,[22] in which Player 1 makes the offer $(x_1^{3*}, 1-x_1^{3*})$. So, Player 2's equilibrium payoff by making the offer $(x_2^2, 1-x_2^2)$ will be $1-x_1^{3*} = U_2^R(x_1^1, x_2^2, x_1^{3*}) = 1 - x_1^1 + \frac{(x_1^1)^2}{2} - \frac{(x_2^2)^2}{2}$,[23] which is decreasing in x_2^2. So, Player 2's optimal offer will be $(0,1)$.

Similarly, we can show that in period 1, Player 1's optimal offer will be $(1,0)$, which Player 2 rejects (the proof is similar to the proof of the optimal offer of Player 2 and is omitted).

It can be verified that $x_1^{3*}(x_1^1, x_2^2) = \frac{1}{2}$ when $x_1^1 = 1$ and $x_2^2 = 0$. We thus have shown that a subgame perfect equilibrium of the game is such that Player 1 makes the offer $(1,0)$ in period 1, which Player 2 rejects, Player 2 makes the offer $(0,1)$ in period 2, which Player 1 rejects, and Player 1 makes the offer $(1/2, 1/2)$ in period 3, which Player 2 accepts. $\square$

Additional Proof of Theorem 1:

We consider the general case where δ is less than but is sufficiently close to one. We focus on the case where $N = 2$. The proof below can be generalized to the case where $N > 2$.

Suppose Player 2 rejects Player 1's offer $(x_1, 1-x_1)$ at Stage 1 and the game moves to Stage 2. Assume that $x_1 > 0$.[24] Let $(x_2, 1-x_2)$ be the offer made by Player 2 at Stage 2. Let $g_1(x_1, x_2) = U_1(x_1)(1 - P(x_2|x_1, x_2)) + U_1(x_2)P(x_2|x_1, x_2)$ be Player 1's arbitration payoff when Player 1's offer is $(x_1, 1-x_1)$ and Player 2's offer is $(x_2, 1-x_2)$. Let $\epsilon(\delta) \in [0, x_1)$ be such that $U_1(x_1 - \epsilon(\delta)) = \delta g_1(x_1, x_1 - \epsilon(\delta))$. For any $\delta \in (0,1]$, $\epsilon(\delta)$ exists and is unique because (i) $U_1(x_1-\epsilon) - \delta g_1(x_1, x_1-\epsilon) = (U_1(x_1) - U_1(x_1-\epsilon))(\delta P(x_1-\epsilon|x_1, x_1-\epsilon) - 1) + (1-\delta)U_1(x_1)$ is continuous and strictly decreasing in ϵ, (ii) when $\epsilon = 0$, we have $U_1(x_1-\epsilon) - \delta g_1(x_1, x_1-\epsilon) \geq 0$ because $U_1(x_1) \geq \delta g_1(x_1, x_1) = \delta U_1(x_1)$, and (iii) when $\epsilon = x_1$, we have $U_1(x_1-\epsilon) - \delta g_1(x_1, x_1-\epsilon) < 0$ because $U_1(0) < \delta g_1(x_1, 0) = \delta(U_1(x_1)(1 - P(0|x_1, 0)) + U_1(0)P(0|x_1, 0))$. In addition, it can be easily verified that $\epsilon(\delta)$ is strictly decreasing in δ and $\epsilon(1) = 0$.

We have the following two cases.

[22] In particular, the acceptance of $(x_2^2, 1-x_2^2)$ will yield a payoff of x_2^2 for Player 1, while rejection of $(x_2^2, 1-x_2^2)$ will lead the game to period 3, in which Player 1 obtains an equilibrium payoff of x_1^{3*}, which is higher than x_2^2.

[23] The first equality follows from the definition of x_1^{3*}.

[24] Notice that it is never optimal for Player 1 to make an offer $(x_1, 1-x_1)$ with $x_1 = 0$ at Stage 1.

(i) $x_2 \geq x_1 - \epsilon(\delta)$.

In this case, Player 1 must choose to accept Player 2's offer $(x_2, 1-x_2)$ in equilibrium, and Player 2's maximum (stage-2) payoff is $U_2(x_1 - \epsilon(\delta))$.

(ii) $x_2 < x_1 - \epsilon(\delta)$.

In this case, Player 1 must choose to reject Player 2's offer $(x_2, 1-x_2)$ in equilibrium. Let $g_2(x_1, x_2) = U_2(1-x_1)(1 - P(x_2|x_1, x_2)) + U_2(1-x_2)P(x_2|x_1, x_2)$ be Player 2's arbitration payoff when Player 1's offer is $(x_1, 1-x_1)$ and Player 2's offer is $(x_2, 1-x_2)$. Let $x_2^*(x_1, \delta)$ be such that[25]

$$x_2^*(x_1, \delta) = \underset{x_2 \in [0, x_1 - \epsilon(\delta)]}{argmax} \; g_2(x_1, x_2).$$

Notice that $x_2^*(x_1, \delta)$ is well-defined, because $g_2(x_1, x_2)$ is continuous (this is because f and U_2 are continuous). Notice that $x_2^*(x_1, 1) < x_1$, because $x_1 > 0$ and f is positive on the whole support $[0, 1]$ by assumption. Let $\delta_1^*(x_1)$ be the unique $\delta \in (0, 1)$ such that $x_1 - \epsilon(\delta) = x_2^*(x_1, 1)$. Then, for any $\delta \in (\delta_1^*(x_1), 1]$, we have $x_2^*(x_1, \delta) = x_2^*(x_1, 1)$. Thus, if $\delta \in (\delta_1^*(x_1), 1]$, then Player 2's maximum (stage-2) payoff is $\delta g_2(x_1, x_2^*(x_1, 1))$ in case (ii).

Let $\delta_2^*(x_1)$ be the unique $\delta \in (0, 1)$ such that $U_2(1 - (x_1 - \epsilon(\delta))) = \delta g_2(x_1, x_2^*(x_1, 1))$. If $\delta \in (\max\{\delta_1^*(x_1), \delta_2^*(x_1)\}, 1]$, then $U_2(1 - (x_1 - \epsilon(\delta))) < \delta g_2(x_1, x_2^*(x_1, 1))$ and thus Player 2's optimal action at Stage 2 is to make the offer $(x_2^*(x_1, 1), 1 - x_2^*(x_1, 1))$, which Player 1 chooses to reject in equilibrium.

Now, let's go back to Stage 1. Notice that it is never optimal for Player 1 to make the offer $(x_1, 1-x_1)$ with $x_1 = 0$, because he can obtain a positive payoff by making any offer $(x_1, 1-x_1)$ where $x_1 > 0$. For any Player 1's offer $(x_1, 1-x_1)$ with $x_1 > 0$, if Player 2 accepts the offer, then Player 2's payoff is $U_2(1-x_1)$. If instead, Player 2 rejects the offer, assuming that $\delta \in (\max\{\delta_1^*(x_1), \delta_2^*(x_1)\}, 1]$, then Player 2 will make the offer $(x_2^*(x_1, 1), 1 - x_2^*(x_1, 1))$ at Stage 2 (which Player 1 rejects), and Player 2's payoff is $\delta^2 g_2(x_1, x_2^*(x_1, 1))$. Let $\delta_3^*(x_1)$ be the unique $\delta \in (0, 1)$ such that $U_2(1-x_1) = \delta^2 g_2(x_1, x_2^*(x_1, 1))$, then if $\delta \in (\delta_3^*(x_1), 1]$, we have $U_2(1-x_1) < \delta^2 g_2(x_1, x_2^*(x_1, 1))$. Thus, if $\delta \in (\max\{\delta_1^*(x_1), \delta_2^*(x_1), \delta_3^*(x_1)\}, 1]$, then

[25] We slightly abuse the notation here. Note that there are may be multiple $x_2^*(x_1, \delta)$ that attain the maximum. In that case, $x_2^*(x_1, \delta)$ refers to (any) one of them.

Player 2's optimal action at Stage 1 is to reject Player 1's offer $(x_1, 1-x_1)$. Let x_1^* be such that[26]

$x_1^* = \underset{x_1 \in [0,1]}{argmax}\, g_1(x_1, x_2^*(x_1, 1))$.

Suppose x_1^* exists.[27] Notice that if x_1^* exists, then we must have $x_1^* > 0$. Now, choose any $\eta_1 \in (0,1)$. We can find an $\eta_2 \in (0, x_1^*)$ such that $U_1(x_1^* - \eta_2) < (\eta_1)^2 g_1(x_1^*, x_2^*(x_1^*, 1))$. Let $\delta^* = \max\{\eta_1, \max_{x_1 \in [x_1^* - \eta_2, 1]}\{\max\{\delta_1^*(x_1), \delta_2^*(x_1), \delta_3^*(x_1)\}\}\}$. So, if $\delta \in (\delta^*, 1]$, then (i) it is never optimal for Player 1 to make an offer $(x_1, 1-x_1)$ with $x_1 \in [0, x_1^* - \eta_2]$, and (ii) if Player 1 makes an offer $(x_1, 1-x_1)$ with $x_1 \in (x_1^* - \eta_2, 1]$, then it is optimal for Player 1 to make the offer $(x_1^*, 1-x_1^*)$. As a result, if $\delta \in (\delta^*, 1]$, the optimal offer that Player 1 can make is $(x_1^*, 1-x_1^*)$.

Let $x_2^*(x_1) = x_2^*(x_1, 1)$. Based on the analysis above, when $\delta \in (\delta^*, 1]$, the subgame perfect equilibrium of the final-offer arbitration game is such that at Stage 1, Player 1 makes the offer $(x_1^*, 1-x_1^*)$, which Player 2 rejects, and at Stage 2, Player 2 makes the offer $(x_2^*(x_1), 1-x_2^*(x_1))$, which Player 1 rejects.

Similarly, for the general case where $N > 2$, it can be shown that if players are sufficiently patient, then in equilibrium, players will always reject the opponents' equilibrium offers, and the equilibrium outcome is such that (i) if N is even, then $x_1^{(N)} = x_1^*$ and $x_2^{(N)} = x_2^*(x_1)$, and (ii) if N is odd, then $x_2^{(N)} = x_2^*$ and $x_1^{(N)} = x_1^*(x_2)$, where $x_2^* = \underset{x_2 \in [0,1]}{argmax}\, g_2(x_1^*(x_2), x_2)$ and $x_1^*(x_2) = \underset{x_1 \in [x_2,1]}{argmax}\, g_1(x_1, x_2)$ (noting that the calculations of x_1^*, $x_2^*(x_1)$, x_2^*, and $x_1^*(x_2)$ are all independent of the discount factor δ). $\square$

References

S. J. Brams, S. Merrill III, Equilibrium strategies for final-offer arbitration: there is no median convergence, *Management Science* 29(8) (1983), 927-41.

[26] Note that there are may be multiple x_1^* that attain the maximum. In that case, x_1^* refers to one of them.

[27] A sufficient condition for x_1^* to exist is that $x_2^*(x_1, 1)$ is continuous in x_1 (notice that we can always select $x_2^*(x_1, 1)$ in a way such that $x_2^*(x_1, 1)$ is continuous). If $x_2^*(x_1, 1)$ is such that x_1^* does not exist, then the subgame perfect equilibrium does not exist for $\delta \in (\delta^*, 1]$, where δ^* will be defined shortly.

O. Compte, P. Jehiel, Gradualism in bargaining and contribution games, *Review of Economic Studies* 71 (2004), 975-1000.

C. A. Deck and A. Farmer, Bargaining over an uncertain value: arbitration mechanisms compared, *Journal of Law, Economics, and Organization* 23 (2007), 547-579.

H. S. Farber, An analysis of final-offer arbitration, *Journal of Conflict Resolution* 24(4) (1980), 683-705.

H. S. Farber, Splitting-the-difference in interest arbitration, *Industrial and Labor Relations Review* 35 (1981), 70-77.

H. S. Farber, M. H. Bazerman, The general basis of arbitrator behavior: an empirical analysis of conventional and final-offer arbitration, *Econometrica* 54(4) (1986), 819-844.

R. Fernandez, J. Glazer (2011), Striking for a Bargain Between Two Completely Informed Agents, *American Economic Review* 81 (1) (1991), 240-252.

P. Manzini, M. Mariotti, Perfect equilibria in a model of bargaining with arbitration, *Games and Economic Behavior* 37 (2001), 170195.

P. Manzini, M. Mariotti, Going alone together: joint outside options in bilateral negotiations, *Economic Journal* 114 (2004), 943-960.

C. Ponsatí, J. Sákovics, Rubinstein bargaining with two-sided outside options, *Economic Theory* 11 (1998), 667-672.

K. Rong, Alternating-offer games with final-offer arbitration, *Games and Economic Behavior* 76(2) (2012), 596-610.

A. Rubinstein, Perfect equilibrium in a bargaining model, *Econometrica* 50 (1982), 97-110.

C. Stevens, Is compulsory arbitration compatible with bargaining?, *Industrial Relations* 5 (1966), 38-52.

M. Yildiz, Nash meets Rubinstein in final-offer arbitration, *Economics Letters* 110 (2011), 226-230.