Bargaining with Split-the-Difference Arbitration

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Abstract We analyze an alternating-offer model in which an arbitrator uses the split-the-difference arbitration rule to determine the outcome if both players' offers are rejected by the opponents. We find that the usual chilling effect of split-the-difference arbitration arises only when the discount factor is sufficiently large. When the discount factor is sufficiently small, players tend to reach agreement immediately. When the discount factor is in the middle range, delayed agreements might arise. We also find that as long as players are not excessively impatient, then the player who makes the first offer obtains an equilibrium payoff that is not greater than his opponent.

Keywords: Split-the-difference arbitration; Alternating-offer game.

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1 Introduction

In the bargaining literature about alternating-offer games, there exist two regularities. One is that players tend to reach agreement immediately, especially in the framework of complete information. The other is that the player who proposes first usually obtains a higher equilibrium payoff than the player who proposes second. This is the so-called first-mover advantage. In this paper, we propose a variant of the finite-horizon alternating-offer game where an arbitrator determines the outcome whenever the players cannot reach an agreement by themselves in a given finite periods. This describes the situation where the players have initiated the arbitration procedure while still being able to negotiate with each other before the arbitrator closes the case. We assume that the arbitrator uses the split-the-difference arbitration rule.[1] We find that the two common regularities in the bargaining literature are overturned. In particular, we find that delay in agreement (and even the activation of arbitration) may occur in equilibrium although our model is of complete information. In addition, we find that as long as the players are not excessively impatient, the second proposer obtains a higher equilibrium payoff than the first proposer. So, we obtain the second-mover advantage, rather than the first-mover advantage.

More specifically, the game proposed in this paper is a three-stage game. At the first two stages, two players, Player 1 and Player 2, sequentially make offers. Player 1 makes the first offer. If a player's offer is accepted by the other player, then that offer is the bargaining outcome and the game ends. If both players' offers are rejected by their opponents, then the game moves to the arbitration stage, during which an arbitrator splits the difference between the players' offers.

We find that the equilibrium of the game depends on the discount factor. In particular, (i) when the discount factor is sufficiently small, the equilibrium features immediate agreement. (ii) When the discount factor is sufficiently large, the equilibrium features arbitration. That

[1] Split-the-difference arbitration is widely used in the real world. For example, NHL salary disputes are required to be resolved by arbitration, and in practice, the arbitrator usually splits the difference between the disputed offers (see, for example, http://nypost.com/2009/08/03/rangers-near-z-day/).

is, both players' offers are rejected, and the arbitration outcome is the final outcome. (iii) When the discount factor is not too small and not too large, delayed agreement might arise in equilibrium.[2]

In our model, the arbitration service is activated in equilibrium only when both players are sufficiently *patient*. This is due to the so-called chilling effect. That is, the players have incentives to make demands that are as extreme as possible, because the time cost of going to arbitration is small, and under the split-the-difference arbitration rule, a player's arbitration payoff increases as the player's demand increases. On the other hand, if players are sufficiently *impatient*, then the players tend to reach agreements immediately in order to avoid the time cost of delay.

An interesting result is that when the players' discount factor is in some middle range, delays in agreements may arise. That is, in equilibrium, Player 1 makes an offer that Player 2 *rejects*, and Player 2 makes a counteroffer that Player 1 *accepts*. In the standard alternating-offer game with no arbitration (e.g., Rubinstein (1982)), delay does not occur because given a strategy profile in which delay occurs, Player 1 can always propose the outcome expected by Player 2, and Player 2 will accept such an offer in order to avoid the time cost of delay. However, in our model, if Player 1 proposes this expected outcome to Player 2, then Player 2 may have an incentive to opt for arbitration. In particular, Player 2 may have an incentive to reject Player 1's offer and move the game to a stage which is closer to arbitration. Notice that if the discount factor is too small, then this threat of going to arbitration becomes non-credible. If the discount factor is too large, then after rejecting Player 1's offer, Player 2 may prefer to make an extreme demand rather than make an offer that will be accepted by Player 1. So, delay in agreement may arise in equilibrium only when the discount factor is in some middle range.

Bargaining models with complete information usually lead to no delay in equilibrium (e.g., Binmore et al. (1989); Ståhl (1972); Rubinstein (1982)). However, delay in bargaining

[2]In this "middle" range, immediate-agreement equilibrium may also arise.

is also a common empirical phenomenon. Many explanations of delay in bargaining have been proposed in the bargaining literature. They include the introduction of asymmetric information over players' valuations (e.g., Cramton (1984, 1992) and Feinberg and Skrzypacz (2005)), the existence of exogenous irrational types (Abreu and Gul (2000)), restrictions on timing of offers (Admati and Perry (1987)), and the multiplicity of the equilibria in the model (Manzini and Mariotti (2001, 2004), Ponsatí and Sákovics (1998)). Our paper provides a new perspective. That is, delay in bargaining may emerge when there exists the threat of going to arbitration during the bargaining process.

We also find that, as long as players are not excessively impatient, then Player 1 obtains an equilibrium payoff that is not greater than the equilibrium payoff of Player 2. This is the case even when the equilibrium is an immediate-agreement equilibrium. This implies that Player 1 suffers as the first mover. This result is in sharp contrast with the standard bargaining theory, in which Player 1, as the first mover, usually obtains a higher equilibrium payoff than his opponent. In our model, Player 2 has a second-mover advantage because Player 2 can make his offer at a stage which is closer to arbitration than Player 1, and thus Player 2 can more credibly threaten to move the game to arbitration than Player 1. Notice that this second-mover advantage of Player 2 occurs only when the players are not excessively impatient. When players are impatient, the time cost of going to arbitration is high, and the threat of going to arbitration will become uncredible.

The classic bargaining theory features the first-mover advantage (e.g., Rubinstein (1982)). However, it is also well-known that if players are allowed to have different discount factors and if the second mover is sufficiently more patient than the first mover, then the second-mover advantage may arise in the infinite-horizon alternating-offer model. In a recent paper, Miettinen and Perea (2013) consider a finite-horizon alternating-offer model where players can commit to some share of the pie prior to each bargaining round at some small cost. They show that the second-mover advantage occurs when the horizon goes to infinity and the commitment cost goes to zero. Our model differs from the literature by offering a new mechanism for explaining the second-mover advantage by introducing arbitration into the bargaining game.

Anbarci and Boyd (2011) also consider a bargaining model in which the arbitrator splits the difference when players are not able to reach agreement by themselves. Our model differs from Anbarci and Boyd (2011) in the sense that Anbarci and Boyd (2011) consider a *simultaneous-offer* model with split-the-difference arbitration, while we consider an *alternating-offer* model with split-the-difference arbitration. While using the simultaneous-offer model usually results in multiple equilibria, the advantage of using the alternating-offer model is that the equilibrium of the game is usually unique.

Several other closely related paper are Rong (2012a), Yildiz (2011) and Rong (2012b). All these papers consider a finite-horizon alternating-offer model that involves arbitration. However, the arbitration procedures used in these papers are different from ours. In particular, Rong (2012a) uses the symmetric arbitration solution, which is an axiomatic arbitration solution.[3] Yildiz (2012) uses the Nash final-offer arbitration rule, in which the arbitrator's ideal settlement is the Nash bargaining solution and the arbitrator chooses the offer that is closest to the Nash bargaining solution as the outcome. Rong (2012b) considers a general class of final-offer arbitration rules, in which the arbitrator's ideal settlement can be any point on the Pareto frontier.

This paper is organized as follows. Section 2 defines the "split-the-difference alternating-offer game." Section 3 analyzes the equilibrium behavior of the split-the-difference alternating-offer game. Concluding remarks are offered in Section 4.

[3]Although the symmetric arbitration solution and the split-the-difference arbitration rule coincide with each other when the Pareto frontier is linear, Rong (2012a) focuses on the case where both players are sufficiently patient, while this paper studies the general case where the discount factor can be any value between 0 and 1.

2 The model

Two players, Players 1 and 2, are bargaining over a unit of money. We assume that both players' utilities are linear in money. The bargaining set (which is the set of the players' feasible utility payoffs) is normalized to be $S = \{(x, y) : x \geq 0, y \geq 0, x + y \leq 1\}$. The Pareto frontier of the bargaining set S is $PF = \{(x, y) : x + y = 1\}$. We define $f(x) = 1 - x$ for $x \in [0, 1]$. Then, $(x, y) \in PF$ if and only if $y = f(x)$. We use $(x_1, y_1) \in S$ to denote Player 1's offer and use $(x_2, y_2) \in S$ to denote Player 2's offer, where x represents Player 1's utility payoff and y represents Player 2's utility payoff. For simplicity, we assume that players can only make offers on the Pareto frontier.

This paper considers the following *split-the-difference alternating-offer game*:

Stage 1: Player 1 makes an offer $(x_1, y_1) \in PF$. Player 2 decides whether to accept the offer, ending the game with (x_1, y_1), or to reject the offer, moving the game to the next stage;

Stage 2: Player 2 makes an offer $(x_2, y_2) \in PF$. Player 1 decides whether to accept the offer, ending the game with (x_2, y_2), or to reject the offer, moving the game to the next stage;

Stage 3: An arbitrator splits the difference between the players' offers. That is, $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$ is the final outcome.

For each player, the payoff obtained at stage i is subject to a discount of δ^{i-1}, where the discount factor $\delta \in (0, 1)$.

3 Analysis

3.1 Equilibrium behavior

This section studies the (subgame-perfect) equilibrium behavior of the split-the-difference alternating-offer game. For any given $(x_1, y_1) \in PF$, define $\hat{x}_2(x_1, y_1) = \frac{\delta}{2 - \delta} x_1$. We have the following lemma.

Lemma 1. *In any equilibrium of the split-the-difference alternating-offer game, if Player 2 rejects Player 1's offer* $(x_1, y_1) \in PF$ *at Stage 1, then at Stage 2, Player 2 must either offer* $(0, 1)$*, which Player 1 rejects, or offer* $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$*, which Player 1 accepts.*

Proof: See the appendix. □

For any given offer $(x_1, y_1) \in PF$ made by Player 1, Player 2 can either make an offer that will be accepted by Player 1, or make an offer that will be rejected by Player 1. Lemma 1 says that in the former case, the best offer that Player 2 can make is $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$, and in the latter case, the best offer that Player 2 can make is $(0, 1)$.

Roughly speaking, Lemma 1 implies that for any offer $(x_1, y_1) \in PF$ made by Player 1, Player 2 can always make two "threats." One threat is to make the counteroffer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$, which Player 1 *accepts*. The other threat is to make the extreme offer $(0, 1)$, which Player 1 *rejects*. In the standard alternating-offer game (e.g., Rubinstein (1982)), it is never optimal for a player to make an offer that will be rejected by the opponent. However, in our model, it is possible that a player makes an extreme offer in equilibrium, which will be rejected by the opponent. The multiple threats facing Player 1 is a key feature of our split-the-difference alternating-offer game. Due to the multiple threats, Player 1's bargaining power in the game is significantly undermined.[4]

Based on Lemma 1, if Player 1 offers $(x_1, y_1) \in PF$ at Stage 1, then in equilibrium, Player 2 must choose one of the following three options:

(A) accepts the offer (x_1, y_1);

(R_c) rejects (x_1, y_1), and at Stage 2, makes the counteroffer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$, which Player 1 accepts;

(R_e) rejects (x_1, y_1), and at Stage 2, makes the extreme offer $(0, 1)$, which Player 1 rejects.

We define the *acceptance region* of Player 2 as the collection of Player 1's offers for which Player 2's best response is to accept (i.e., Player 2's best response is option A). That is, the

[4]We will further illustrate this point in section 3.2.

acceptance region $\overline{A} = \{(x_1, y_1) \in PF | (x_1, y_1)$ is such that $A \succeq_2 R_c$ and $A \succeq_2 R_e\}$, where $X \succeq_2 Y$ means that option X gives Player 2 a payoff that is not less than option Y. Similarly, we define the *weak rejection region* $\overline{R}_c$ as the collection of Player 1's offers for which Player 2's best response is option R_c, and the *strong rejection region* $\overline{R}_e$ as the collection of Player 1's offers for which Player 2's best response is option R_e. That is, $\overline{R}_c = \{(x_1, y_1) \in PF | (x_1, y_1)$ is such that $R_c \succeq_2 A$ and $R_c \succeq_2 R_e\}$ and $\overline{R}_e = \{(x_1, y_1) \in PF | (x_1, y_1)$ is such that $R_e \succeq_2 A$ and $R_e \succeq_2 R_c\}$.

The following lemma characterizes the acceptance region, the weak rejection region and the strong rejection region for any given discount factor $\delta \in (0, 1)$. Define $x_1^*(\delta) = \dfrac{2-\delta}{2+\delta}$, $x_2^*(\delta) = \dfrac{2-2\delta^2}{2-\delta^2}$ and $x_3^*(\delta) = \dfrac{2(2-\delta)(1-\delta)}{\delta^2}$.[5] We have:

Lemma 2.

(i) For $\delta \in (0.868, 1)$*, we have* $\overline{A} = \{(x_1, y_1) \in PF | x_1 \in [0, x_2^*(\delta)]\}$*,* $\overline{R}_c = \emptyset$*, and* $\overline{R}_e = \{(x_1, y_1) \in PF | x_1 \in [x_2^*(\delta), 1]\}$*;*

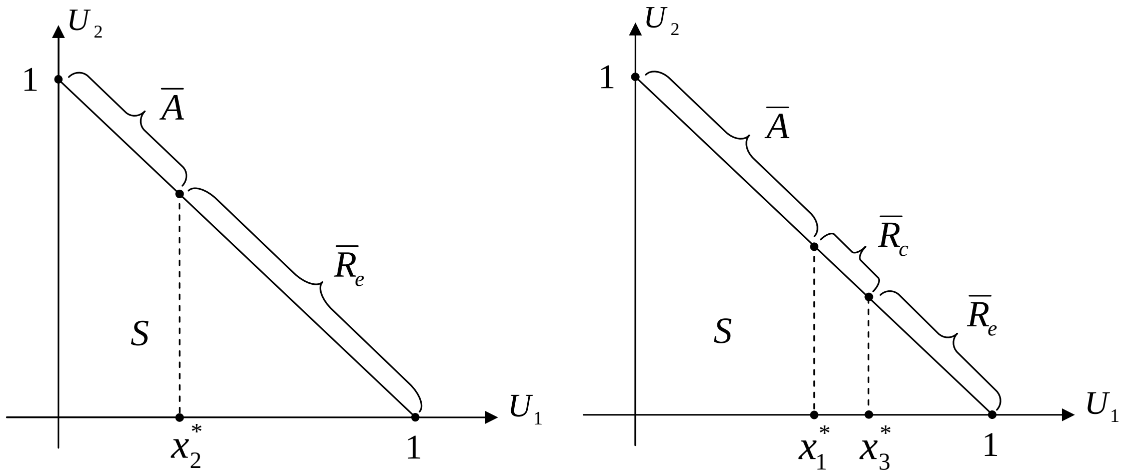
(ii) For $\delta \in [0.763, 0.868]$*, we have* $\overline{A} = \{(x_1, y_1) \in PF | x_1 \in [0, x_1^*(\delta)]\}$*,* $\overline{R}_c = \{(x_1, y_1) \in PF | x_1 \in [x_1^*(\delta), x_3^*(\delta)]\}$*, and* $\overline{R}_e = \{(x_1, y_1) \in PF | x_1 \in [x_3^*(\delta), 1]\}$*;*

(iii) For $\delta \in (0, 0.763)$*, we have* $\overline{A} = \{(x_1, y_1) \in PF | x_1 \in [0, x_1^*(\delta)]\}$*,* $\overline{R}_c = \{(x_1, y_1) \in PF | x_1 \in [x_1^*(\delta), 1]\}$*, and* $\overline{R}_e = \emptyset$*.*

Proof: See the appendix. □

Figure 1 and Figure 2 depict the acceptance region, the weak rejection region and the strong rejection region for the three cases listed in Lemma 2. In Lemma 2, there are two thresholds of the discount factor: 0.868 and 0.763. The first threshold 0.868 is the unique solution to $x_1^*(\delta) = x_2^*(\delta) = x_3^*(\delta)$. If $\delta > 0.868$, then $x_3^*(\delta) < x_2^*(\delta) < x_1^*(\delta)$. If $\delta < 0.868$,

[5] $(x_1^*(\delta), f(x_1^*(\delta)))$ is such that Player 2 is indifferent between option A and option R_c. $(x_2^*(\delta), f(x_2^*(\delta)))$ is such that Player 2 is indifferent between option A and option R_e. $(x_3^*(\delta), f(x_3^*(\delta)))$ is such that Player 2 is indifferent between option R_c and option R_e.

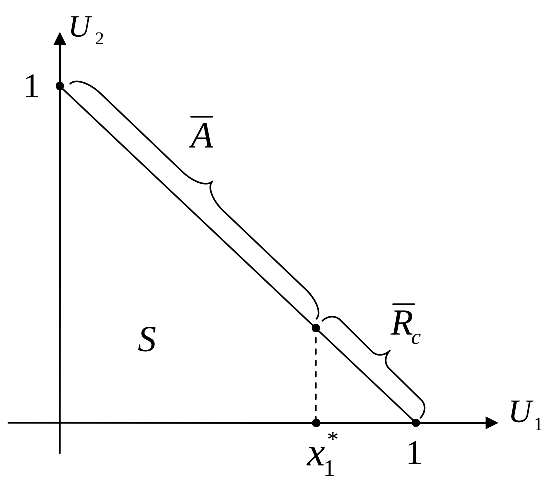


Case 1: $\delta \in (0.868,1)$

Case 2: $\delta \in [0.763,0.868]$

Figure 1

then $x_1^*(\delta) < x_2^*(\delta) < x_3^*(\delta)$. The second threshold 0.763 is the unique solution to $x_3^*(\delta) = 1$. It can be verified that $x_3^*(\delta) < 1$ if and only if $\delta > 0.763$.



Case 3: $\delta \in (0,0.763)$

Figure 2

Notice that when δ decreases, $x_1^*(\delta)$, $x_2^*(\delta)$ and $x_3^*(\delta)$ all increase. This suggests the following two observations: (i) the acceptance region increases as δ decreases and the acceptance region approaches the entire Pareto frontier as δ goes to 0,[6] and (ii) the strong rejection region decreases as δ decreases and the strong rejection region totally vanishes as δ becomes

[6]Use the fact that $x_1^* \to 1$ as $\delta \to 0$.

smaller than 0.763. Observation (i) reflects the fact that as players become sufficiently *impatient*, players tend to reach agreement immediately in order to avoid the time cost of going to arbitration. Observation (ii) implies that the chilling effect of split-the-difference arbitration exists only when the players are sufficiently *patient*, i.e., $\delta > 0.763$. If, instead, $\delta < 0.763$, then it is never optimal for Player 2 to reject Player 1's offer and make the extreme demand.

For any given Player 1's offer $(x_1, y_1) \in PF$, Player 2 chooses either option A, or option R_c, or option R_e. The corresponding payoffs of Player 1 are x_1, $\frac{\delta^2}{2-\delta}x_1$ and $\delta^2 \frac{x_1}{2}$ respectively. In all the three cases, the payoff of Player 1 is strictly increasing in x_1. This implies that Player 1 will never make an offer that is strictly inside the acceptance region, or the weak rejection region, or the strong rejection region. In other words, Player 1's equilibrium offer must be either $(x_1^*(\delta), f(x_1^*(\delta)))$, or $(x_2^*(\delta), f(x_2^*(\delta)))$, or $(x_3^*(\delta), f(x_3^*(\delta)))$, or $(1, 0)$.[7] This result is further illustrated in the following lemma. Notice that in the remainder of the paper, for simplicity, I write $x_1^*(\delta)$, $x_2^*(\delta)$ and $x_3^*(\delta)$ as x_1^*, x_2^* and x_3^* respectively.

Lemma 3.

(i) If $\delta \in (0.868, 1)$*, then in equilibrium, Player 1 makes either the offer* $(x_2^*, f(x_2^*))$ *or the offer* $(1, 0)$*;*

(ii) If $\delta \in [0.763, 0.868]$*, then in equilibrium, Player 1 makes either the offer* $(x_1^*, f(x_1^*))$*, or the offer* $(x_3^*, f(x_3^*))$*, or the offer* $(1, 0)$*;*

(iii) If $\delta \in (0, 0.763)$*, then in equilibrium, Player 1 makes either the offer* $(x_1^*, f(x_1^*))$ *or the offer* $(1, 0)$*.*

Now, we can state the main result of our equilibrium analysis. We show in Theorem 4 that three types of equilibria appear as the discount factor varies from 1 to 0. In Theorem 4, an equilibrium with *immediate agreement* is one in which Player 1 makes an offer that Player 2 accepts immediately. An equilibrium with *delayed agreement* is one in which Player 1 makes

[7] The offer $(0, 1)$ is also not strictly inside the acceptance region or the weak/strong rejection region. However, $(0, 1)$ cannot be Player 1's equilibrium offer because $(0, 1)$ is strictly dominated by the offer $(1, 0)$ and thus Player 1 will never make the offer $(0, 1)$ in equilibrium.

an offer that Player 2 rejects; and at the next stage, Player 2 makes a counteroffer that Player 1 accepts. An equilibrium with *no agreement* is one in which all offers are rejected and the final outcome splits the difference between offers.

In the proof of Theorem 4, we need the following two tie-breaking rules to simplify the analysis.

Tie-breaking rule 1: If a player is indifferent between acceptance and rejection, then he chooses acceptance.

Tie-breaking rule 2: If a player is indifferent between two options that he offers his opponent, then he chooses the option that yields his opponent a higher payoff.

Theorem 4. *The equilibrium of the split-the-difference alternating-offer game as a function of δ is described by Table 1.*

δ	Equilibrium Type	Equilibrium Initial Offer (x_1)	Equilibrium Counteroffer (x_2)
$0.874 < \delta < 1$	No agreement	1	0
$0.868 < \delta \leq 0.874$	Immediate agreement	x_2^*	NA
$0.781 \leq \delta \leq 0.868$	Immediate agreement	x_1^*	NA
$0.763 \leq \delta < 0.781$	Delayed agreement	x_3^*	$\delta x_3^*/(2-\delta)$
$0.752 < \delta < 0.763$	Delayed agreement	1	$\delta/(2-\delta)$
$0 < \delta \leq 0.752$	Immediate agreement	x_1^*	NA

Table 1: The equilibrium of the split-the-difference alternating-offer game.

Proof:

We have the following three cases.

Case 1: $\delta > 0.868$.

In this case, according to Lemma 3, Player 1 makes either the offer $(x_2^*, f(x_2^*))$, or the offer $(1, 0)$ in equilibrium. Player 1's payoff is either x_2^*, or $\frac{\delta^2}{2}$. It can be verified that $x_2^* \geq \frac{\delta^2}{2}$ if and only if $\delta \leq 0.874$. Therefore, if $0.868 < \delta \leq 0.874$, then Player 1 makes the offer $(x_2^*, f(x_2^*))$, which Player 2 accepts.[8] If $\delta > 0.874$, then Player 1 makes the offer $(1, 0)$, which

[8]Actually, if Player 1 makes the offer $(x_2^*, f(x_2^*))$, then Player 2 is indifferent between acceptance and rejection. However, using tie-breaking rule 1, Player 2 chooses to accept the offer $(x_2^*, f(x_2^*))$.

Player 2 rejects and makes the counteroffer $(0,1)$ (which Player 1 rejects).

Case 2: $0.763 \leq \delta \leq 0.868$.

In this case, according to Lemma 3, Player 1 makes either the offer $(x_1^*, f(x_1^*))$, or the offer $(x_3^*, f(x_3^*))$, or the offer $(1,0)$. Player 1's payoff is either x_1^*, or $2(1-\delta)$, or $\frac{\delta^2}{2}$. It can be verified that when $0.763 \leq \delta < 0.781$, we have $2(1-\delta) > x_1^*$ and $2(1-\delta) > \frac{\delta^2}{2}$. Thus, Player 1 makes the offer $(x_3^*, f(x_3^*))$, which Player 2 rejects and makes the counteroffer $(\frac{\delta}{2-\delta}x_3^*, 1-\frac{\delta}{2-\delta}x_3^*)$ which Player 1 accepts.[9] If, instead, $0.781 \leq \delta \leq 0.868$, then $x_1^* \geq 2(1-\delta)$ and $x_1^* \geq \frac{\delta^2}{2}$. Thus, Player 1 makes the offer $(x_1^*, f(x_1^*))$, which Player 2 accepts.

Case 3: $\delta < 0.763$.

In this case, according to Lemma 3, Player 1 makes either the offer $(x_1^*, f(x_1^*))$, or the offer $(1,0)$. Player 1's payoff is either x_1^*, or $\frac{\delta^2}{2-\delta}$. It can be verified that $x_1^* \geq \frac{\delta^2}{2-\delta}$ if and only if $\delta \leq 0.752$. Thus, if $0 < \delta \leq 0.752$, Player 1 makes the offer $(x_1^*, f(x_1^*))$, which Player 2 accepts. If, instead, $0.752 < \delta < 0.763$, then Player 1 makes the offer $(1,0)$, which Player 2 rejects and makes the counteroffer $(\frac{\delta}{2-\delta}, 1-\frac{\delta}{2-\delta})$ which Player 1 accepts. □

Theorem 4 shows that when the discount factor is small ($\delta \leq 0.752$), the equilibrium features immediate agreement. When the discount factor is sufficiently large ($\delta > 0.874$), the equilibrium features no agreement. When the discount factor is in the middle range ($0.752 < \delta \leq 0.874$), the equilibrium is either an equilibrium with immediate agreement, or an equilibrium with delayed agreement.

The players use the arbitration service only when they are sufficiently patient ($\delta > 0.874$). In addition, in this case, both players make the extreme demands. This is because the more a player demands, the more payoff he obtains during the arbitration stage.

When the discount factor is in the middle range, both the immediate-agreement equilibrium and the delayed-agreement equilibrium may arise. The intuition that the delayed

[9] Actually, Player 2 is indifferent between making the counteroffer $(\frac{\delta}{2-\delta}x_3^*, 1-\frac{\delta}{2-\delta}x_3^*)$ and making the extreme offer. Using tie-breaking rule 2, Player 2 makes the counteroffer $(\frac{\delta}{2-\delta}x_3^*, 1-\frac{\delta}{2-\delta}x_3^*)$ which Player 1 accepts.

agreement arises can be explained by Figure 3 (for the case where $\delta \in (0.752, 0.763)$).[10] When $\delta \in (0.752, 0.763)$, according to Lemma 2, $\overline{A} = [0, x_1^*]$ and $\overline{R}_c = [x_1^*, 1]$. Therefore, if Player 1 makes an offer that Player 2 accepts, then his best option is to offer $(x_1^*, f(x_1^*))$. Player 1's payoff is thus x_1^*. If, instead, Player 1 makes an offer that Player 2 rejects, then his best option is to offer $(1, 0)$, which Player 2 rejects and makes the counteroffer $(\hat{x}_2(1, 0), f(\hat{x}_2(1, 0)))$, which Player 1 accepts. Player 1's payoff is thus $\delta\hat{x}_2(1, 0)$. When $\delta \in (0.752, 0.763)$, it can be verified that $x_1^* = \dfrac{2-\delta}{2+\delta} < \delta\hat{x}_2(1, 0) = \delta\dfrac{\delta}{2-\delta}$. So, Player 1 prefers to make the offer $(1, 0)$ (which Player 2 rejects), and delayed agreement occurs in equilibrium.

Since players reach an agreement at $(\hat{x}_2(1, 0), f(\hat{x}_2(1, 0)))$ at Stage 2, one may wonder why they cannot reach such an agreement at Stage 1? The key to understand the answer to the question is to notice that in the standard alternating-offer model, players' bargaining behavior is *history-independent*, while in our model, players' bargaining behavior is *history-dependent*. That is, in our model, Player 1's initial offer may affect the equilibrium counteroffer of Player 2, because the potential arbitration outcome is not an exogenous outcome, but is endogenously determined by both players' offers. When Player 1 makes the offer $(1, 0)$ at Stage 1, Player 2's optimal counteroffer at Stage 2 is $(\hat{x}_2(1, 0), f(\hat{x}_2(1, 0)))$. However, when Player 1 makes the offer $(\hat{x}_2(1, 0), f(\hat{x}_2(1, 0)))$ at Stage 1, Player 2's optimal counteroffer at Stage 2 is another counteroffer $(\hat{x}_2(\hat{x}_2(1, 0)), f(\hat{x}_2(\hat{x}_2(1, 0))))$. Moreover, the discounted value of this new counteroffer is better than $(\hat{x}_2(1, 0), f(\hat{x}_2(1, 0)))$ for Player 2. So, Player 2 has an incentive to reject $(\hat{x}_2(1, 0), f(\hat{x}_2(1, 0)))$, rather than accept it at Stage 1.

3.2 Equilibrium payoffs

Based on the players' equilibrium strategies listed in Table 1, we depict the equilibrium payoff received by Player 1 in Figure 4 (refer to the solid line in Figure 4). An interesting result in Figure 4 is that, for some ranges of discount factors, as the discount factor increases,

[10] For the case where $\delta \in [0.763, 0.781)$, the graphic explanation is similar.

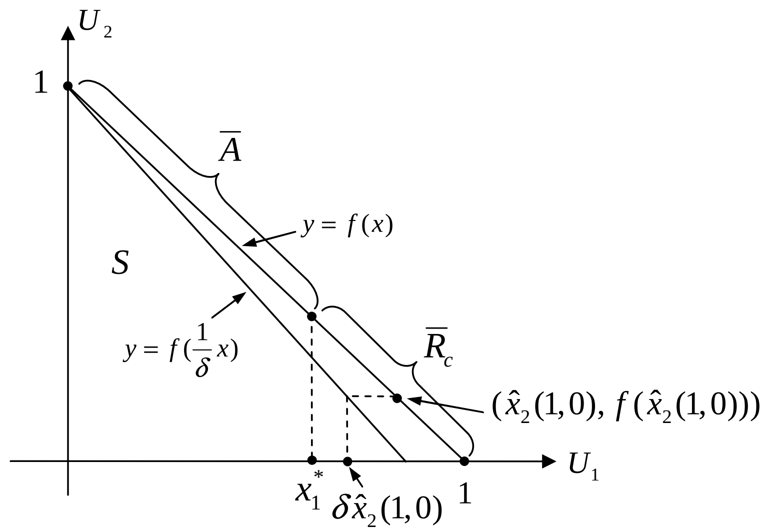


Figure 3: The case where $\delta \in (0.752, 0.763)$

the equilibrium payoff of Player 1 increases.[11] This happens when the equilibrium is either an equilibrium with delayed agreement ($0.752 < \delta < 0.763$), or an equilibrium with no agreement ($0.874 < \delta < 1$). In those two types of equilibria, no agreement is reached at Stage 1. Thus, in those two equilibria, after Player 2 rejects Player 1's offer, the game moves to Stage 2 and Player 2 becomes the proposer. The switch of the proposer role between the two players complicates the relationship between the discount factor and the initial proposer's equilibrium payoff and makes it possible for Player 1 to increase his equilibrium payoff as the discount factor rises. In contrast, in the standard alternating-offer model that features immediate agreement (e.g., Rubinstein, 1982), as players become more patient, the payoff obtained by Player 1 decreases.

Figure 4 also shows that Player 1's equilibrium payoff is consistently less than the Rubinstein equilibrium payoff.

Next, we compare Player 1's equilibrium payoff and Player 2's equilibrium payoff. As shown in the next theorem, we find that as long as players are not excessively impatient, then Player 1 obtains an equilibrium payoff that is *not greater than* the equilibrium payoff of Player 2.

[11]Notice that Player 1's payoff obtained from the Rubinstein equilibrium is strictly decreasing in δ for $\delta \in (0, 1)$.

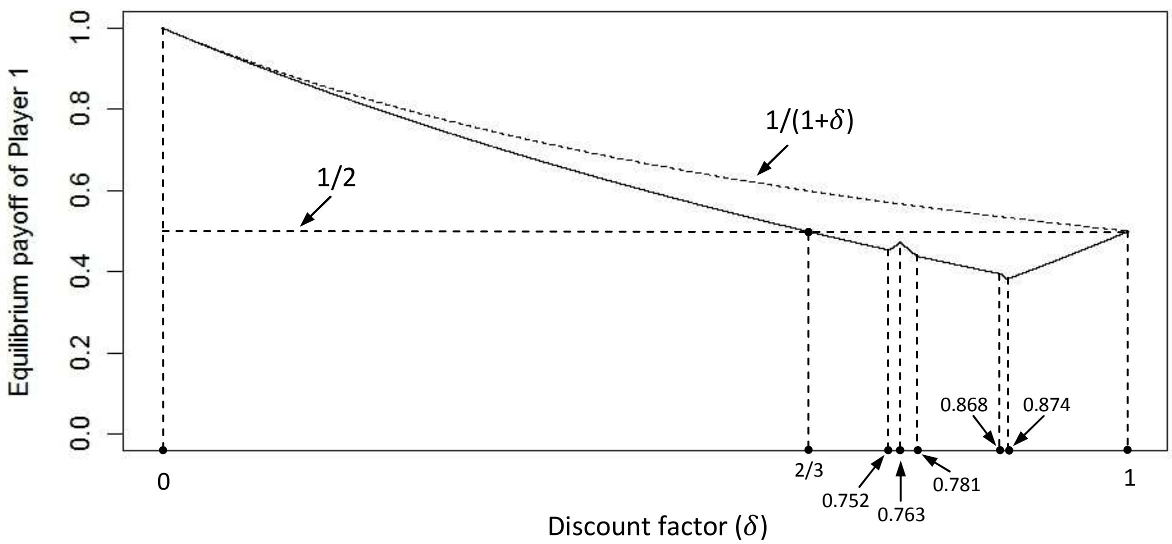


Figure 4: Equilibrium payoff to Player 1 as a function of the discount factor.

Let $EP_1(\delta)$ and $EP_2(\delta)$ denote the equilibrium payoffs of Player 1 and Player 2 respectively. We have:

Theorem 5. *$EP_1(\delta) \leq EP_2(\delta)$ for any $\delta \in [0.781, 1)$.*

Proof: See the appendix. □

Theorem 5 stands in contrast to the standard bargaining theory (Rubinstein, 1982), which predicts that Player 1 has the first-mover advantage and is able to extract more surplus than Player 2. In our game, as long as players are not excessively impatient, then Player 1 suffers as the first mover. That is, Player 1's bargaining power is "less" than Player 2's bargaining power. The reason is as follows. As in the standard alternating-offer model, Player 1 (as the player who makes the first offer) has the first-mover advantage because both players prefer to reach agreement immediately in order to avoid the time cost of delay. On the other hand, Player 2 has the second-mover advantage of being "closer" to the arbitration stage, and thus Player 2 can more credibly threaten to move the game to arbitration than Player 1. Roughly speaking, when the players become more and more patient, Player 1's first-mover advantage decreases, while Player 2's second-mover advantage increases. It is thus not surprising that when players are not excessively impatient, Player 1's bargaining power is "less" than Player 2's, except the extreme case where δ is close to 1. When δ is close to 1, both players have incentives to make extreme demands due to the chilling effect of split-the-difference arbitration, and the equilibrium payoffs for the two players equal (this implies that the two players' relative bargaining power equals as δ approaches 1).

4 Conclusions

This paper studies a finite-horizon alternating-offer model that involves split-the-difference arbitration. We find that the unique SPE of the game depends on the discount factor. When the discount factor is small, players reach agreement immediately. When the discount factor is large, players make extreme demands and the arbitrator splits the difference between the players' extreme offers. When the discount factor is in the middle range, delayed agreements can arise, i.e., players reach agreement at the second stage. We also find that as long as players are not excessively impatient, then Player 1 suffers as the first mover.

An interesting extension of our model is to consider *unequal* split-the-difference arbitration. In particular, the arbitrator can put different weights on the two players' offers, where the weight on one player's offer depends on the distance between the player's offer and the arbitrator's ideal settlement.[12] As a player's offer becomes relatively more distant from the arbitrator's ideal settlement, the weight on the player's offer decreases. This endogenous unequal split-the-difference arbitration rule will give players some incentives to moderate their positions. As a result, the usual chilling effect of split-the-difference arbitration will become less severe, and players may tend to reach more agreements by themselves under unequal split-the-difference arbitration.

This paper assumes that players have risk-neutral preferences. In an earlier version of the paper (Rong (2011)), we find that most of the results obtained is this paper are robust to the introduction of risk aversion. In particular, we find that if players are sufficiently patient,

[12]See also Farber (1981) for a similar analysis in the setting where the two players make simultaneous offers.

then players will make extreme demands due to the chilling effect of split-the-difference arbitration, and if players are sufficiently impatient, then players will reach immediate agreement in order to avoid the time cost of delay.

Risk aversion can also help us to better understand the motivation of the paper. In this paper, we assume that the game will move to the arbitration stage if the two players cannot reach agreement by themselves. A natural question is, what will happen if the players are allowed to choose whether to go to arbitration before bargaining begins? This paper finds that if players agree to arbitration before bargaining begins, then compared with the bargaining game with no arbitration, the second mover enjoys an increased bargaining power because the second-mover can more credibly threaten to move the game to arbitration (this is particularly true when the discount factor is large). This implies that the imbalance of the bargaining power between the first mover and the second mover is smaller in the bargaining game with arbitration than in the game with no arbitration. So, assuming that players are risk averse and that the players' proposing order is random, it may be optimal for players to agree to arbitration even if the players are allowed to choose to not have arbitration.

Appendix

Proof of Lemma 1:

Using the definition of $\hat{x}_2(x_1, y_1)$, we have $\delta\hat{x}_2 = \delta^2\frac{\hat{x}_2 + x_1}{2}$. This implies that, if Player 2 rejects Player 1's offer $(x_1, f(x_1))$ at Stage 1 and proposes the counteroffer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$ at Stage 2, then Player 1 must be indifferent between accepting the counteroffer and rejecting it (see Figure 5). In fact, observing that $\delta\hat{x}_2 \geq \delta^2\frac{\hat{x}_2 + x_1}{2}$ if and only if $x_2 \geq \hat{x}_2(x_1, y_1)$, Player 1 will accept Player 2's offer (x_2, y_2) at Stage 2 if and only if $x_2 \geq \hat{x}_2(x_1, y_1)$. Thus, if Player 2 wants to make an offer that Player 1 accepts, then his best option is to offer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$. In addition, if Player 2 wants to make an offer that Player 1 rejects, then his best option is to make the extreme offer (i.e., $(0, 1)$),

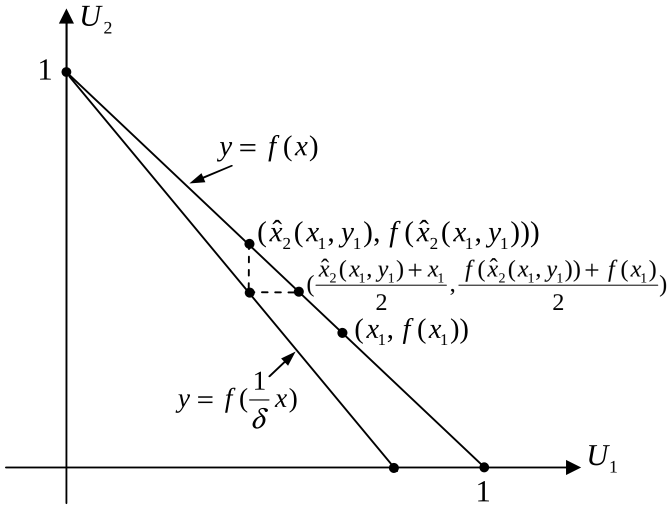


Figure 5: Definition of $\hat{x}_2(x_1, y_1)$.

because the arbitrated payoff $\frac{y_1 + y_2}{2}$ received by Player 2 is strictly increasing in y_2. $\square$

Proof of Lemma 2:

Note that if Player 1 offers $(x_1, y_1) \in PF$ at Stage 1, then Player 2 either accepts it (option A), or rejects it with the counteroffer $(\frac{\delta}{2-\delta}x_1, 1 - \frac{\delta}{2-\delta}x_1)$ that Player 1 accepts (option R_c), or rejects it with the counteroffer $(0, 1)$ that Player 1 rejects (option R_e). The corresponding payoffs are x_1, $\frac{\delta^2}{2-\delta}x_1$ and $\delta^2\frac{x_1}{2}$ for Player 1, and $1 - x_1$, $\delta(1 - \frac{\delta}{2-\delta}x_1)$ and $\delta^2\frac{2-x_1}{2}$ for Player 2.

Figure 6, Figure 7 and Figure 8 depict the payoff of Player 2 as a function of x_1 for Player 2's three options. For Player 2, option A and option R_c become indifferent when $x_1 = \frac{2-\delta}{2+\delta} = x_1^*$. Option A and option R_e become indifferent when $x_1 = \frac{2-2\delta^2}{2-\delta^2} = x_2^*$. Option R_c and option R_e become indifferent when $x_1 = \frac{2(2-\delta)(1-\delta)}{\delta^2} = x_3^*$. We have the following three cases.

Case 1 (refer to Figure 6): $x_2^* < x_1^*$, i.e., $\delta > 0.868$.

In this case, if Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \leq x_2^*$, then Player 1 will choose option A. If Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \geq x_2^*$, then Player 1 will choose option R_e.[13]

[13]When $x_1 = x_2^*$, Player 1 is indifferent between option A and option R_e.

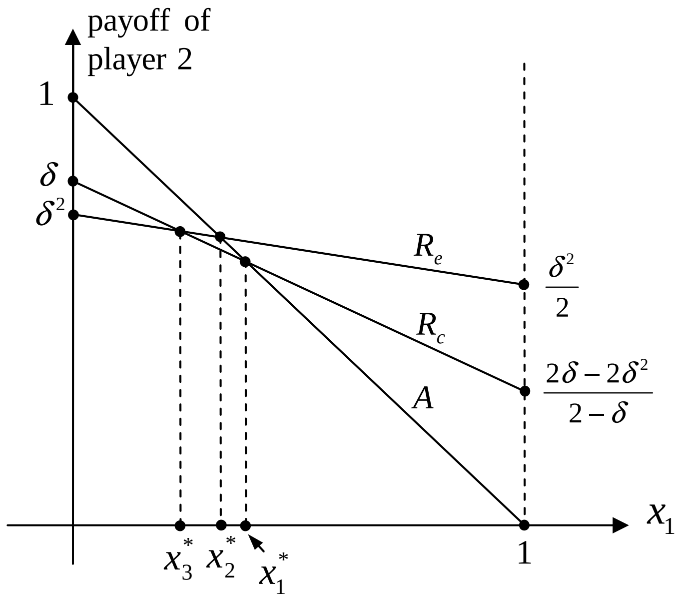


Figure 6: Case 1.

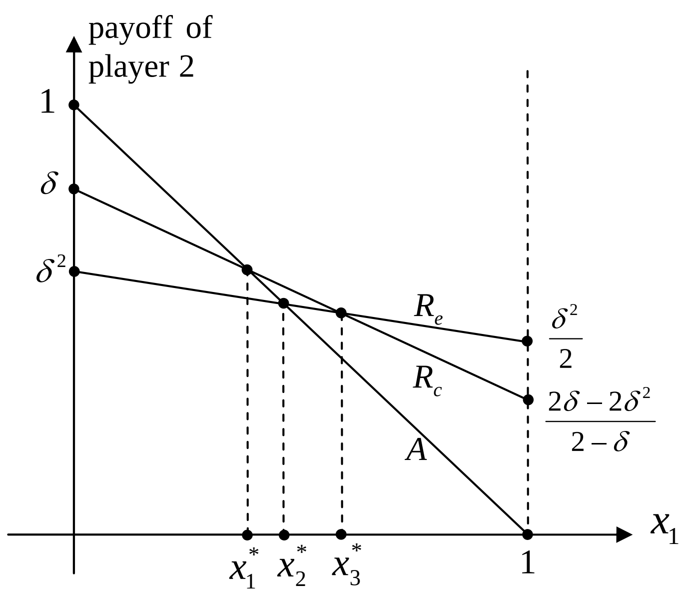


Figure 7: Case 2.

Case 2 (refer to Figure 7): $x_2^* \geq x_1^*$ and $x_3^* \leq 1$, i.e., $0.763 \leq \delta \leq 0.868$.

In this case, if Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \leq x_1^*$, then Player 1 will choose option A. If Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1^* \leq x_1 \leq x_3^*$, then Player 1 will choose option R_c. If Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \geq x_3^*$, then Player 1 will choose option R_e.

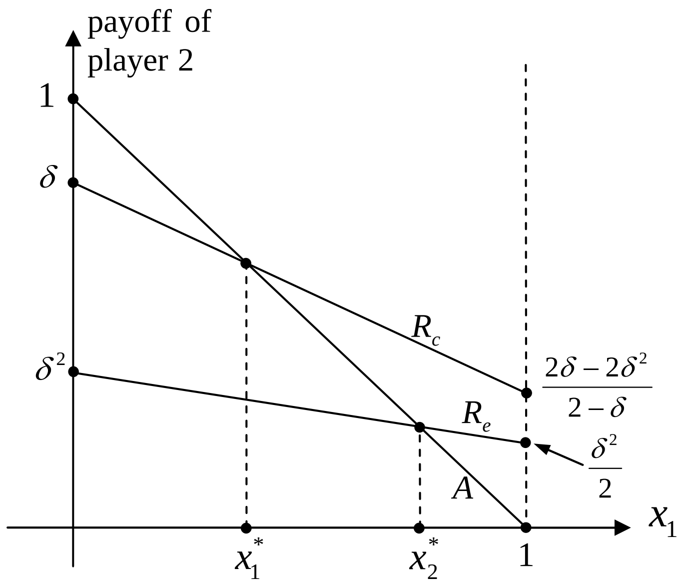


Figure 8: Case 3.

Case 3 (refer to Figure 8): $x_3^* \geq 1$, i.e., $\delta \leq 0.763$.

In this case, if Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \leq x_1^*$, then Player 1 will choose option A. If Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \geq x_1^*$, then Player 1 will choose option R_c. $\square$

Proof of Theorem 5:

Using Table 1, one can show that $EP_1(\delta) = \dfrac{2-\delta}{2+\delta}$ and $EP_2(\delta) = \dfrac{2\delta}{2+\delta}$ for $\delta \in [0.781, 0.868]$, $EP_1(\delta) = \dfrac{2-2\delta^2}{2-\delta^2}$ and $EP_2(\delta) = \dfrac{\delta^2}{2-\delta^2}$ for $\delta \in (0.868, 0.874]$, and $EP_1(\delta) = EP_2(\delta) = \dfrac{\delta^2}{2}$ for $\delta \in (0.874, 1)$. It is easy to verify that $EP_1(\delta) \leq EP_2(\delta)$ for any $\delta \in [0.781, 1)$. $\square$

References

D. Abreu, F. Gul, *Bargaining and reputation*, Econometrica 68 (2000), 85-117.

A. R. Admati, M. Perry, *Strategic delay in bargaining*, Review of Economic Studies 54 (1987), 345-364.

N. Anbarci, J. H. Boyd, *Nash demand game and the Kalai-Smorodinsky solution*, Games and Economic Behavior 71 (2011), 14-22.

K. Binmore, A. Shaked, J. Sutton, *An outside option experiment*, Quarterly Journal of Economics 104 (1989), 753-770.

P. C. Cramton, *Bargaining with incomplete information: an infinite-horizon model with two-side uncertainty*, Review of Economic Studies 51 (1984), 579-593.

P. C. Cramton, *Strategic delay in bargaining with two-sided uncertainty*, Review of Economic Studies 59 (1992), 205-225.

H. S. Farber, *Splitting-the-difference in interest arbitration*, Industrial and Labor Relations Review 35 (1981), 70-77.

Y. Feinberg and A. Skrzypacz, *Uncertainty about uncertainty and delay in bargaining*, Econometrica 73 (2005), 69-91.

P. Manzini, M. Mariotti, *Perfect equilibria in a model of bargaining with arbitration*, Games Economic Behavior 37 (2001), 170195.

P. Manzini, M. Mariotti, *Going alone together: joint outside options in bilateral negotiations*, Economic Journal 114 (2004), 943960.

T. Miettinen, A. Perea, *Commitment in alternating-offers bargaining*, working paper, 2013.

C. Ponsatí, J. Sákovics, *Rubinstein bargaining with two-sided outside options*, Economic Theory 11 (1998), 667-672.

K. Rong, *An axiomatic approach to arbitration and its application in bargaining games*, The B.E. Journal of Theoretical Economics (Contributions) 12 (2012), Article 28.

K. Rong, *Alternating-offer games with final-offer arbitration*, Games and Economic Behavior 76 (2012), 596-610.

K. Rong, *Bargaining with split-the-difference arbitration*, working paper, 2011.

A. Rubinstein, *Perfect equilibrium in a bargaining model*, Econometrica 50 (1982), 97-110.

I. Ståhl, *Bargaining theory*, Stockholm: Economics Research Institute, Stockholm School of Economics (1972).

M. Yildiz, *Nash meets Rubinstein in final-offer arbitration*, Economics Letters 110 (2011), 226-230.