Bargaining with Split-the-Difference Arbitration

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Abstract We analyze an alternating-offer model in which an arbitrator uses the split-the-difference arbitration rule to determine the outcome if both players' offers are rejected by the opponents. We find that the usual chilling effect of split-the-difference arbitration only arises when the discount factor is sufficiently large. When the discount factor is sufficiently small, players tend to reach agreement immediately. When the discount factor is in the middle range, delayed agreements might arise. We also find that as long as players are not excessively impatient, then the player who makes the first offer obtains an equilibrium payoff that is not greater than his opponent.

Keywords: Split-the-difference; Alternating-offer game.

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1 Introduction

Alternating-offer game is a common bargaining procedure. In the literature, the research about the alternating-offer game usually focuses on the case where the players can bargain with each other for (possibly) infinite periods until an agreement is reached. This paper assumes that players can only bargain with each other for finite periods[1] and if an agreement is not reached, then an arbitrator is called in. We assume that the arbitrator uses the split-the-difference arbitration rule to decide the outcome. The purpose of our paper is to explore the impact of the introduction of split-the-difference arbitration on players' equilibrium behavior and equilibrium payoffs in the alternating-offer game. More particularly, our paper addresses the following questions: (i) do players actually use the arbitration service in equilibrium, or do they prefer to reach agreements by themselves? and, (ii) which player benefits from the introduction of split-the-difference arbitration? That is, which player obtains a higher equilibrium payoff?

The game proposed in this paper is a simple three-stage game. At the first two stages, two players, Player 1 and Player 2, sequentially make offers. Player 1 makes the first offer. If a player's offer is accepted by the other player, then that offer is the bargaining outcome and the game ends. If both players' offers are rejected by their opponents, then the game moves to the arbitration stage, during which an arbitrator splits the difference between the players' offers.

We find that the equilibrium of the game depends on the discount factor. In particular, (i) when the discount factor is sufficiently small, the equilibrium features immediate agreement. (ii) When the discount factor is sufficiently large, the equilibrium features arbitration. That is, both players' offers are rejected, and the arbitration outcome is the final outcome. (iii) When the discount factor is not too small and not too large, delayed agreement might arise in

[1]Although the model in this paper assumes that players can only bargain with each other for two periods, the model can be easily extended to the case where the players can bargain with each other for any given finite periods.

equilibrium.[2] We also find that, as long as players are not excessively impatient, then Player 1 obtains an equilibrium payoff that is not greater than the equilibrium payoff received by Player 2. This is the case even when the equilibrium is an immediate-agreement equilibrium.

In our model, the arbitration-type equilibrium appears only when both players are sufficiently *patient*. This is natural because if players are impatient, then the players tend to reach agreements by themselves in order to avoid the time cost of going to arbitration. An interesting result is that when the players' discount factor is in some middle range, delayed agreements may arise in equilibrium. That is, in equilibrium, Player 1 makes an offer that Player 2 *rejects*, and Player 2 makes a counteroffer that Player 1 *accepts*. The intuition of this result is as follows. If the game moves to the arbitration stage, then the split-the-difference outcome depends on both players' offers. This implies that if Player 2 chooses to reject Player 1's offer at Stage 1, then Player 2's optimal counteroffer at Stage 2 will depend on Player 1's offer. In addition, for any given Player 1's offer that Player 2 chooses to reject, the optimal counteroffer that Player 2 makes is closer to Player 1's initial offer as the players become patient[3] (this is true as long as the players are not too patient, in which case both players tend to make the extreme demands due to the so-called chilling effect). It turns out that when the discount factor is in some middle range, Player 1 prefers to make a high demand that Player 2 chooses to reject,[4] because the counteroffer that Player 2 makes is close to Player 1's offer and the discounted value of the counteroffer is better than any other offer that Player 1 could make and Player 2 accepts.

We also find that, as long as players are not excessively impatient, then Player 1 obtains an equilibrium payoff that is not greater than the equilibrium payoff of Player 2. This is the case even when the equilibrium is an immediate-agreement equilibrium. This implies that Player 1 suffers as the first mover. This result is in sharp contrast with the standard

[2]In this "middle" range, immediate-agreement equilibrium may also arise.

[3]This is because as players become patient, the time cost of going to arbitration is small and Player 2 has to offer a more favorable counteroffer to Player 1 in order to induce Player 1 to accept it.

[4]Player 1's offer will be rejected by Player 2 if and only if Player 1's demand is higher than a threshold demand.

bargaining theory (e.g., Rubinstein, 1982), in which Player 1, as the first mover, usually obtains a higher equilibrium payoff than his opponent. In our model, Player 1 suffers because if Player 1's offer is not generous enough to Player 2, then Player 2 can threaten to reject Player 1's offer and make the extreme demand (which Player 2 rejects and the arbitrator splits the difference between the players' offers). This threat of Player 2 is credible when players are patient. This additional threat[5] of Player 2 undermines Player 1's bargaining power and makes Player 1 obtain an equilibrium payoff that is not greater than Player 2 when both players are patient.

Anbarci and Boyd (2011) also consider a bargaining model in which the arbitrator splits the difference when players are not able to reach agreement by themselves. Our model differs from Anbarci and Boyd (2011) in the sense that Anbarci and Boyd (2011) consider a *simultaneous-offer* model with split-the-difference arbitration, while we consider an *alternating-offer* model with split-the-difference arbitration. While using the simultaneous-offer model usually results in multiple equilibria, the advantage of using the alternating-offer model is that the equilibrium of the game is usually unique.

Several other closely related paper are Rong (2012a), Yildiz (2011) and Rong (2012b). All these papers consider a finite-horizon alternating-offer model that involves arbitration. However, the arbitration procedures used in these papers are different from the split-the-difference arbitration rule used in this paper. In particular, Rong (2012a) uses the symmetric arbitration solution, which is an axiomatic arbitration solution.[6] Yildiz (2012) uses the Nash final-offer arbitration rule, in which the arbitrator's ideal settlement is the Nash bargaining solution and the arbitrator chooses the offer that is closest to the Nash bargaining solution as the outcome. Rong (2012b) considers a general class of final-offer arbitration rules, in

[5]Besides this threat, Player 2 can also threaten to reject Player 1's offer and make an offer that is less generous than Player 1's offer, but more generous than the extreme offer. Player 2 accepts such an offer in order to avoid the time cost of going to arbitration.

[6]Although the symmetric arbitration solution and the split-the-difference arbitration rule coincide with each other when the Pareto frontier is linear, Rong (2012a) focuses on the case where both players are sufficiently patient, while this paper studies the general case where the discount factor can be any number between 0 and 1.

which the arbitrator's ideal settlement can be any point on the Pareto frontier.

This paper is organized as follows. Section 2 defines the "split-the-difference alternating-offer game." Section 3 analyzes the equilibrium behavior of the split-the-difference alternating-offer game. Concluding remarks are offered in Section 4.

2 The model

Two players, Players 1 and 2, are bargaining over a unit of money. We assume that both players' utilities are linear in money, and that the bargaining set (which is the set of the players' feasible expected utility payoffs) is $S = \{(x, y) : x \geq 0, y \geq 0, x + y \leq 1\}$. The Pareto frontier of the bargaining set S is $PF = \{(x, y) : x + y = 1\}$. We define $f(x) = 1 - x$ for $x \in [0, 1]$. Then, $(x, y) \in PF$ if and only if $y = f(x)$. We use $(x_1, y_1) \in S$ to denote Player 1's offer and use $(x_2, y_2) \in S$ to denote Player 2's offer, where x represents Player 1's utility payoff and y represents Player 2's utility payoff. For simplicity, we assume that players can only make offers on the Pareto frontier.[7]

This paper considers the following *split-the-difference alternating-offer game*:

Stage 1: Player 1 makes an offer $(x_1, y_1) \in PF$. Player 2 decides whether to accept the offer, ending the game with (x_1, y_1), or to reject the offer, moving the game to the next stage;

Stage 2: Player 2 makes an offer $(x_2, y_2) \in PF$. Player 1 decides whether to accept the offer, ending the game with (x_2, y_2), or to reject the offer, moving the game to the next stage;

Stage 3: An arbitrator splits the difference between players' offers. That is, $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$ is the final outcome.

[7]This assumption simplifies the equilibrium analysis and is essential for the main result in the paper. If this assumption is dropped, then it can be shown that Player 1 may make an offer that is not on the Pareto frontier in equilibrium (although Player 2 will always make an offer on the Pareto frontier in equilibrium), and some "strange" equilibrium might arise. This assumption seems to be artificial, but it is also natural if we redefine each player's strategy space in the following way. Each player is asked to make a demand, instead of an offer, when it is his turn to make a proposal. The arbitrator then automatically translates each player's demand to an offer on the Pareto frontier.

For each player, the payoff obtained at stage i is subject to a discount of δ^{i-1}, where the discount factor $\delta \in (0,1)$.

3 Analysis

3.1 Equilibrium behavior

This section studies the equilibrium behavior of the split-the-difference alternating-offer game. For any given $(x_1, y_1) \in PF$, define $\hat{x}_2(x_1, y_1) = \dfrac{\delta}{2-\delta} x_1$. We have the following lemma.

Lemma 1. *In the equilibrium of the split-the-difference alternating-offer game, if Player 2 rejects Player 1's offer $(x_1, y_1) \in PF$ at Stage 1, then at Stage 2, Player 2 must either offer $(0,1)$, which Player 1 rejects, or offer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$, which Player 1 accepts.*

Proof: See the appendix. □

Lemma 1 implies that for any offer $(x_1, y_1) \in PF$ made by Player 1, Player 2 can always make two threats. One threat is the counteroffer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$, which Player 1 *accepts*. The other threat is the extreme offer $(0,1)$, which Player 1 *rejects*. The multiple threats facing Player 1 is a key feature of the split-the-difference alternating-offer game. Due to the multiple threats, Player 1's bargaining power in the game is significantly undermined.[8]

Based on Lemma 1, if Player 1 offers $(x_1, y_1) \in PF$ at Stage 1, then in equilibrium, Player 2 must choose one of the following three options:

(A) accepts the offer (x_1, y_1);

(R_c) rejects (x_1, y_1), and at Stage 2, makes the counteroffer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$, which Player 1 accepts;

(R_e) rejects (x_1, y_1), and at Stage 2, makes the extreme offer $(0,1)$, which Player 1 rejects.

[8] We will further illustrate this point in section 3.2.

We define the *acceptance region* of Player 2 as the collection of Player 1's offers for which Player 2 chooses to accept (i.e., Player 2 chooses option A). That is, the acceptance region $\overline{A} = \{(x_1, y_1) \in PF | (x_1, y_1)$ is such that $A \succeq_2 R_c$ and $A \succeq_2 R_e\}$. Similarly, we define the *weak rejection region* $\overline{R}_c$ as the collection of Player 1's offers for which Player 2 chooses option R_c, and the *strong rejection region* $\overline{R}_e$ as the collection of Player 1's offers for which Player 2 chooses option R_e. That is, $\overline{R}_c = \{(x_1, y_1) \in PF | (x_1, y_1)$ is such that $R_c \succeq_2 A$ and $R_c \succeq_2 R_e\}$ and $\overline{R}_e = \{(x_1, y_1) \in PF | (x_1, y_1)$ is such that $R_e \succeq_2 A$ and $R_e \succeq_2 R_c\}$.

The following lemma characterizes the acceptance region, the weak rejection region and the strong rejection region for any given discount factor $\delta \in (0, 1)$. Define $x_1^*(\delta) = \dfrac{2-\delta}{2+\delta}$, $x_2^*(\delta) = \dfrac{2-2\delta^2}{2-\delta^2}$ and $x_3^*(\delta) = \dfrac{2(2-\delta)(1-\delta)}{\delta^2}$.[9] We have:

Lemma 2.

(i) For $\delta \in (0.868, 1)$*, we have* $\overline{A} = \{(x_1, y_1) \in PF | x_1 \in [0, x_2^*(\delta)]\}$*,* $\overline{R}_c = \emptyset$*, and* $\overline{R}_e = \{(x_1, y_1) \in PF | x_1 \in [x_2^*(\delta), 1]\}$*;*

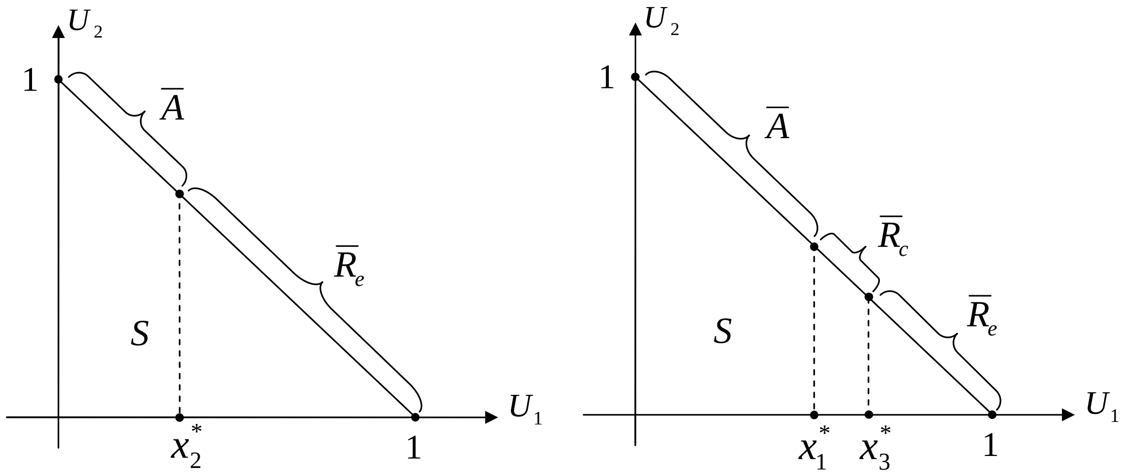
(ii) For $\delta \in [0.763, 0.868]$*, we have* $\overline{A} = \{(x_1, y_1) \in PF | x_1 \in [0, x_1^*(\delta)]\}$*,* $\overline{R}_c = \{(x_1, y_1) \in PF | x_1 \in [x_1^*(\delta), x_3^*(\delta)]\}$*, and* $\overline{R}_e = \{(x_1, y_1) \in PF | x_1 \in [x_3^*(\delta), 1]\}$*;*

(iii) For $\delta \in (0, 0.763)$*, we have* $\overline{A} = \{(x_1, y_1) \in PF | x_1 \in [0, x_1^*(\delta)]\}$*,* $\overline{R}_c = \{(x_1, y_1) \in PF | x_1 \in [x_1^*(\delta), 1]\}$*, and* $\overline{R}_e = \emptyset$*.*

Proof: See the appendix. □

Figure 1 and Figure 2 depict the acceptance region, the weak rejection region and the strong rejection region for the three cases listed in Lemma 2. In Lemma 2, there are two thresholds of the discount factor: 0.868 and 0.763. The threshold 0.868 is the unique solution to $x_1^*(\delta) = x_2^*(\delta)$. If $\delta > 0.868$, then $x_3^*(\delta) < x_2^*(\delta) < x_1^*(\delta)$. If $\delta < 0.868$, then $x_1^*(\delta) < x_2^*(\delta) <$

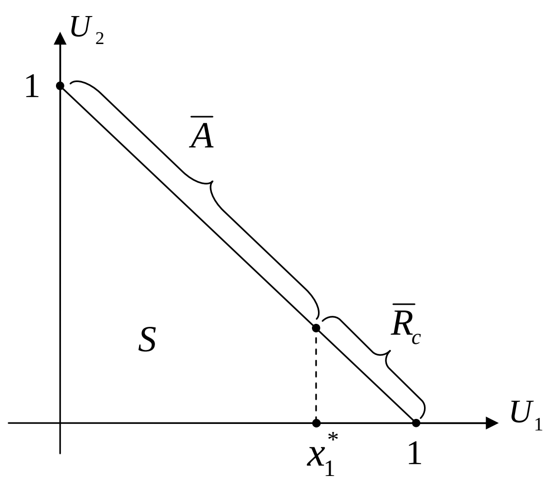
[9] $(x_1^*(\delta), f(x_1^*(\delta)))$ is such that Player 2 is indifferent between option A and option R_c. $(x_2^*(\delta), f(x_2^*(\delta)))$ is such that Player 2 is indifferent between option A and option R_e. $(x_3^*(\delta), f(x_3^*(\delta)))$ is such that Player 2 is indifferent between option R_c and option R_e.



Case 1: $\delta \in (0.868,1)$ Case 2: $\delta \in [0.763,0.868]$

Figure 1

$x_3^*(\delta)$. The other threshold 0.763 is the unique solution to $x_3^*(\delta) = 1$. It can be verified that $x_3^*(\delta) < 1$ if and only if $\delta > 0.763$.



Case 3: $\delta \in (0,0.763)$

Figure 2

Notice that when δ decreases, $x_1^*(\delta)$, $x_2^*(\delta)$ and $x_3^*(\delta)$ all increase. This suggests the following two observations: (i) the acceptance region increases as δ decreases and the acceptance region approaches the entire Pareto frontier as δ goes to 0,[10] and (ii) the strong rejection region decreases as δ decreases and the strong rejection region totally vanishes as δ becomes

[10]Use the fact that $x_1^* \to 1$ as $\delta \to 0$.

smaller than 0.763. Observation (i) reflects the fact that as players become *impatient*, players tend to reach agreement immediately in order to avoid the time cost of going to arbitration. Observation (ii) implies that the chilling effect of split-the-difference arbitration exists only when the players are sufficiently *patient* ($\delta > 0.763$). If, instead, $\delta < 0.763$, then it is never optimal for Player 2 to reject Player 1's offer and make the extreme demand.

For any given Player 1's offer $(x_1, y_1) \in PF$, Player 2 chooses either option A, or option R_c, or option R_e. The corresponding payoffs of Player 1 are x_1, $\frac{\delta^2}{2-\delta}x_1$ and $\delta^2\frac{x_1}{2}$ respectively. In all the three cases, the payoff of Player 1 is striclty increasing in x_1. This implies that Player 1 will never make an offer that is strictly inside the acceptance region, or the weak rejection region, or the strong rejection region. In other words, Player 1's equilibrium offer must be either $(x_1^*(\delta), f(x_1^*(\delta)))$, or $(x_2^*(\delta), f(x_2^*(\delta)))$, or $(x_3^*(\delta), f(x_3^*(\delta)))$, or $(1, 0)$.[11] This result is further illustrated in the following lemma.

Lemma 3.

(i) If $\delta \in (0.868, 1)$*, then in equilibrium, Player 1 makes either the offer* $(x_2^*, f(x_2^*))$ *or the offer* $(1, 0)$*;*

(ii) If $\delta \in [0.763, 0.868]$*, then in equilibrium, Player 1 makes either the offer* $(x_1^*, f(x_1^*))$*, or the offer* $(x_3^*, f(x_3^*))$*, or the offer* $(1, 0)$*;*

(iii) If $\delta \in (0, 0.763)$*, then in equilibrium, Player 1 makes either the offer* $(x_1^*, f(x_1^*))$ *or the offer* $(1, 0)$*.*

Now, we can state the main result of our equilibrium analysis. We show in Theorem 4 that three types of equilibria appear as the discount factor varies from 1 to 0. In Theorem 4, an equilibrium with *immediate agreement* is one in which Player 1 makes an offer that Player 2 accepts immediately. An equilibrium with *delayed agreement* is one in which Player 1 makes an offer that Player 2 rejects; and at the next stage, Player 2 makes a counteroffer that

[11] The offer $(0, 1)$ is also not strictly inside the acceptance region or the weak/strong rejection region. However, $(0, 1)$ cannot be Player 1's equilibrium payoff because the offer $(0, 1)$ is strictly dominated by the offer $(1, 0)$ and thus Player 1 will never make the offer $(0, 1)$ in equilibrium.

Player 1 accepts. An equilibrium with *no agreement* is one in which all offers are rejected and the final outcome splits the difference between offers.

In the proof of Theorem 4, we need the following two tie-breaking rules to simplify the analysis.

Tie-breaking rule 1: If a player is indifferent between acceptance and rejection, then he chooses acceptance.

Tie-breaking rule 2: If a player is indifferent between two options that he offers his opponent, then he chooses the option that yields his opponent a higher payoff.

Theorem 4. *The equilibrium of the split-the-difference alternating-offer game as a function of δ is described by Table 1.*

δ	Equilibrium Type	Equilibrium Initial Offer (x_1)	Equilibrium Counteroffer (x_2)
$0.874 < \delta < 1$	No agreement	1	0
$0.868 < \delta \leq 0.874$	Immediate agreement	x_2^*	NA
$0.781 \leq \delta \leq 0.868$	Immediate agreement	x_1^*	NA
$0.763 \leq \delta < 0.781$	Delayed agreement	x_3^*	$\delta x_3^*/(2-\delta)$
$0.752 < \delta < 0.763$	Delayed agreement	1	$\delta/(2-\delta)$
$0 < \delta \leq 0.752$	Immediate agreement	x_1^*	NA

Table 1: SPE of the split-the-difference alternating-offer game.

Proof:

We have the following three cases.

Case 1: $\delta > 0.868$.

In this case, according to Lemma 3, Player 1 makes either the offer $(x_2^*, f(x_2^*))$, or the offer $(1, 0)$ in equilibrium. Player 1's payoff is either x_2^*, or $\dfrac{\delta^2}{2}$. It can be verified that $x_2^* \geq \dfrac{\delta^2}{2}$ if and only if $\delta \leq 0.874$. Therefore, if $0.868 < \delta \leq 0.874$, then Player 1 makes the offer $(x_2^*, f(x_2^*))$, which Player 2 accepts.[12] If $\delta > 0.874$, then Player 1 makes the offer $(1, 0)$, which Player 2 rejects and makes the counteroffer $(0, 1)$.

[12]Actually, if Player 1 makes the offer $(x_2^*, f(x_2^*))$, then Player 2 is indifferent between acceptance and rejection. However, using tie-breaking rule 1, Player 2 chooses to accept the offer $(x_2^*, f(x_2^*))$.

Case 2: $0.763 \leq \delta \leq 0.868$.

In this case, according to Lemma 3, Player 1 makes either the offer $(x_1^*, f(x_1^*))$, or the offer $(x_3^*, f(x_3^*))$, or the offer $(1, 0)$. Player 1's payoff is either x_1^*, or $2(1-\delta)$, or $\frac{\delta^2}{2}$. It can be verified that when $0.763 \leq \delta < 0.781$, we have $2(1-\delta) > x_1^*$ and $2(1-\delta) > \frac{\delta^2}{2}$. Thus, Player 1 makes the offer $(x_1, y_1) \in PF$ with $x_1 = x_3^*$, which Player 2 rejects and makes the counteroffer $(\frac{\delta}{2-\delta}x_3^*, 1 - \frac{\delta}{2-\delta}x_3^*)$ which Player 1 accepts.[13] If, instead, $0.781 \leq \delta \leq 0.868$, then $x_1^* \geq 2(1-\delta)$ and $x_1^* \geq \frac{\delta^2}{2}$. Thus, Player 1 makes the offer $(x_1, y_1) \in PF$ with $x_1 = x_1^*$, which Player 2 accepts.

Case 3: $\delta < 0.763$.

In this case, according to Lemma 3, Player 1 makes either the offer $(x_1^*, f(x_1^*))$, or the offer $(1, 0)$. Player 1's payoff is either x_1^*, or $\frac{\delta^2}{2-\delta}$. It can be verified that $x_1^* \geq \frac{\delta^2}{2-\delta}$ if and only if $\delta \leq 0.752$. Thus, if $0 < \delta \leq 0.752$, Player 1 makes the offer $(x_1, y_1) \in PF$ with $x_1 = x_1^*$, which Player 2 accepts. If, instead, $0.752 < \delta < 0.763$, then Player 1 makes the offer $(x_1, y_1) \in PF$ with $x_1 = 1$, which Player 2 rejects and makes the counteroffer $(\frac{\delta}{2-\delta}, 1 - \frac{\delta}{2-\delta})$ which Player 1 accepts. □

Theorem 4 shows that when the discount factor is small ($\delta \leq 0.752$), the equilibrium features immediate agreement. When the discount factor is sufficiently large ($\delta > 0.874$), the equilibrium features no agreement. When the discount factor is in the middle range ($0.752 < \delta \leq 0.874$), the equilibrium is either an equilibrium with immediate agreement, or an equilibrium with delayed agreement.

The players use the arbitration service only when they are sufficiently patient ($\delta > 0.874$). In addition, in this case, both players make the extreme demands. This is because the more a player demand, the more payoff he obtains during the arbitration stage.

When the discount factor is in the middle range, both the immediate-agreement equi-

[13] Actually, Player 2 is indifferent between making the counteroffer $(\frac{\delta}{2-\delta}x_3^*, 1 - \frac{\delta}{2-\delta}x_3^*)$ and making the extreme offer. Using tie-breaking rule 2, Player 2 makes the counteroffer $(\frac{\delta}{2-\delta}x_3^*, 1 - \frac{\delta}{2-\delta}x_3^*)$ which Player 1 accepts.

librium and the delayed-agreement equilibrium may arise. The intuition that the delayed agreement arises can be explained by Figure 3 (for the case where $\delta \in (0.752, 0.763)$).[14] When $\delta \in (0.752, 0.763)$, according to Lemma 2, $\overline{A} = [0, x_1^*]$ and $\overline{R}_c = [x_1^*, 1]$. Therefore, if Player 1 makes an offer that Player 2 accepts, then his best option is to offer $(x_1^*, f(x_1^*))$. Player 1's payoff is thus x_1^*. If, instead, Player 1 makes an offer that Player 2 rejects, then his best option is to offer $(1, 0)$,[15] which Player 2 rejects and makes the counteroffer $(\hat{x}_2(1,0), f(\hat{x}_2(1,0)))$, which Player 1 accepts. Player 1's payoff is thus $\delta\hat{x}_2(1,0)$. When $\delta \in (0.752, 0.763)$, it can be verified that $x_1^* = \dfrac{2-\delta}{2+\delta} < \delta\hat{x}_2(1,0) = \delta\dfrac{\delta}{2-\delta}$. So, Player 1 prefers to make the offer $(1, 0)$ (which Player 2 rejects), and delayed agreement occurs in equilibrium.

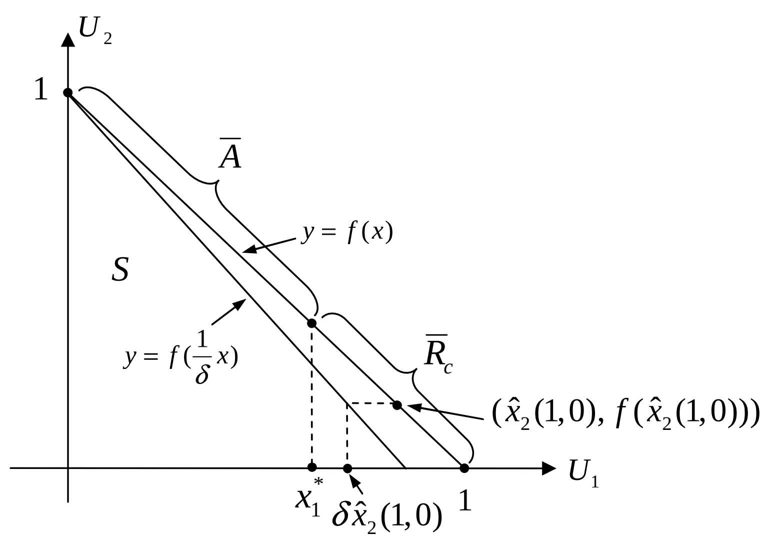


Figure 3: The case where $\delta \in (0.752, 0.763)$

3.2 Equilibrium payoffs

Based on the players' equilibrium strategies listed in Table 1, we depict the equilibrium payoff received by Player 1 in Figure 4. An interesting result in Figure 4 is that, for some

[14]For the case where $\delta \in [0.763, 0.781)$, the graphic explanation is similar.

[15]Notice that the counteroffer of Player 2 $(\frac{\delta}{2-\delta}x_1, f(\frac{\delta}{2-\delta}x_1))$ is more favorable to Player 1 as Player 1's demand x_1 increases.

ranges of discount factors, as the discount factor increases, the equilibrium payoff of Player 1 increases.[16] This happens when the equilibrium is either an equilibrium with delayed agreement ($0.752 < \delta < 0.763$), or an equilibrium with no agreement ($0.874 < \delta < 1$). In those two types of equilibria, no agreement is reached at Stage 1. Thus, in those two equilibria, after Player 2 rejects Player 1's offer, the game moves to Stage 2 and Player 2 becomes the proposer. The switch of the proposer role between the two players complicates the relationship between the discount factor and the initial proposer's equilibrium payoff and makes it possible for Player 1 to increase his equilibrium payoff as the discount factor rises. In contrast, in the standard alternating-offer model that features immediate agreement (e.g., Rubinstein, 1982), as players become more patient, the payoff obtained by Player 1 decreases.

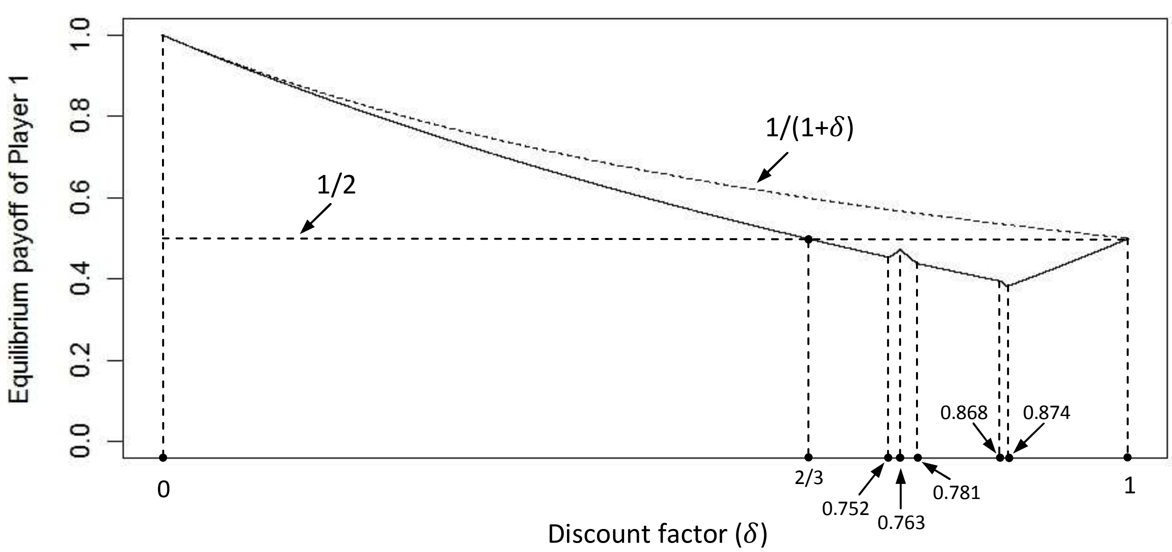


Figure 4: Equilibrium payoff to Player 1 as a function of the discount factor.

Figure 4 also shows that Player 1's equilibrium payoff is consistently less than the Rubinstein equilibrium payoff.

Next, we compare Player 1's equilibrium payoff and Player 2's equilibrium payoff. As shown in the next theorem, we find that as long as players are not excessively impatient,

[16]Notice that Player 1's payoff obtained from the Rubinstein equilibrium is strictly decreasing in δ for $\delta \in (0, 1)$.

then Player 1 obtains an equilibrium payoff that is *not greater than* the equilibrium payoff of Player 2.

Let $EP_1(\delta)$ and $EP_2(\delta)$ denote the equilibrium payoffs of Player 1 and Player 2 respectively. We have:

Theorem 5. *$EP_1(\delta) \leq EP_2(\delta)$ for any $\delta \in [0.781, 1)$.*

Proof: See the appendix. □

Theorem 5 stands in contrast to the standard bargaining theory (Rubinstein, 1982), which predicts that Player 1 has the first-mover advantage and is able to extract more surplus than Player 2. In our game, as long as players are not excessively impatient, then Player 1 suffers as the first mover. That is, Player 1's bargaining power is "less" than Player 2's bargaining power. The reason is as follows. As in the standard alternating-offer model, Player 1 (as the player who makes the first offer) has the first-mover advantage because both players prefer to reach agreement immediately in order to avoid the time cost. On the other hand, Player 2 has the second-mover advantage of being "closer" to the arbitration stage, and thus Player 2 can more credibly threaten to make the extreme demand than Player 1. When the players become more and more patient, Player 1's first-mover advantage decreases, while Player 2's second-mover advantage increases. It is thus not surprising that when players are not excessively impatient, Player 1's bargaining power is "less" than Player 2's.

4 Conclusions

This paper studies a finite-horizon alternating-offer model that involves split-the-difference arbitration. We find that the unique SPE of the game depends on the discount factor. When the discount factor is small, players reach agreement immediately. When the discount factor is large, players make extreme demands and the arbitrator splits the difference between the players' extreme offers. When the discount factor is in the middle range, delayed agreements can arise, i.e., players reach agreement at the second stage. We also find that as long as players are not excessively impatient, then Player 1 suffers as the first mover.

Although this paper focuses on the case where the arbitration rule is *equally* splitting the difference, the main result of the paper can be extended to the general case where the arbitration rule is *unequally* splitting the difference (that is, the arbitrator puts different weights on the two players' offers). Another possible extension of the paper is to allow the arbitrator to have an ideal settlement, and the weight on a player's offer depends on the distance between the player's offer and the arbitrator's ideal settlement.[17]

Appendix

Proof of Lemma 1:

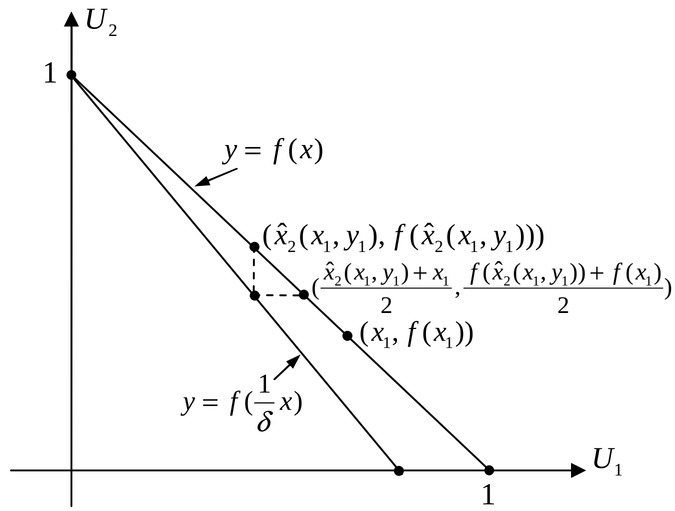


Figure 5: Definition of $\hat{x}_2(x_1, y_1)$.

Using the definition of $\hat{x}_2(x_1, y_1)$, we have $\delta \hat{x}_2 = \delta^2 \dfrac{\hat{x}_2 + x_1}{2}$. This implies that, if Player 2 rejects Player 1's offer $(x_1, f(x_1))$ at Stage 1 and proposes the counteroffer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$ at Stage 2, then Player 1 must be indifferent between accepting the counteroffer and rejecting it (see Figure 5). In fact, observing that $\delta \hat{x}_2 \geq \delta^2 \dfrac{\hat{x}_2 + x_1}{2}$ if

[17] See also Farber (1981) for a similar analysis in the setting where the two players make simultaneous offers.

and only if $x_2 \geq \hat{x}_2(x_1, y_1)$, Player 1 will accept Player 2's offer (x_2, y_2) at Stage 2 if and only if $x_2 \geq \hat{x}_2(x_1, y_1)$. Thus, if Player 2 wants to make an offer that Player 1 accepts, then his best option is to offer $(\hat{x}_2(x_1, y_1), f(\hat{x}_2(x_1, y_1)))$. In addition, if Player 2 wants to make an offer that Player 1 rejects, then his best option is to make the extreme offer (i.e., $(0, 1)$), because the arbitrated payoff $\frac{y_1 + y_2}{2}$ received by Player 2 is strictly increasing in y_2. □

Proof of Lemma 2:

Note that if Player 1 offers $(x_1, y_1) \in PF$ at Stage 1, then Player 2 either accepts it (option A), or rejects it with the counteroffer $(\frac{\delta}{2-\delta}x_1, 1 - \frac{\delta}{2-\delta}x_1)$ that Player 1 accepts (option R_c), or rejects it with the counteroffer $(0, 1)$ that Player 1 rejects (option R_e). The corresponding payoffs are x_1, $\frac{\delta^2}{2-\delta}x_1$ and $\delta^2 \frac{x_1}{2}$ for Player 1, and $1 - x_1$, $\delta(1 - \frac{\delta}{2-\delta}x_1)$ and $\delta^2 \frac{2 - x_1}{2}$ for Player 2.

Figure 6, Figure 7 and Figure 8 depict the payoff of Player 2 as a function of x_1 for Player 2's three options. For Player 2, option A and option R_c become indifferent when $x_1 = \frac{2-\delta}{2+\delta} = x_1^*$. Option A and option R_e become indifferent when $x_1 = \frac{2-2\delta^2}{2-\delta^2} = x_2^*$. Option R_c and option R_e become indifferent when $x_1 = \frac{2(2-\delta)(1-\delta)}{\delta^2} = x_3^*$. We have the following three cases.

Case 1 (refer to Figure 6): $x_2^* < x_1^*$, i.e., $\delta > 0.868$.

In this case, if Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \leq x_2^*$, then Player 1 will choose option A. If Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \geq x_2^*$, then Player 1 will choose option R_e.[18]

Case 2 (refer to Figure 7): $x_2^* \geq x_1^*$ and $x_3^* \leq 1$, i.e., $0.763 \leq \delta \leq 0.868$.

In this case, if Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \leq x_1^*$, then Player 1 will choose option A. If Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1^* \leq x_1 \leq x_3^*$, then Player 1 will choose option R_c. If Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \geq x_3^*$, then Player 1 will choose option R_e.

Case 3 (refer to Figure 8): $x_3^* \geq 1$, i.e., $\delta \leq 0.763$.

[18] When $x_1 = x_2^*$, Player 1 is indifferent between option A and option R_e.

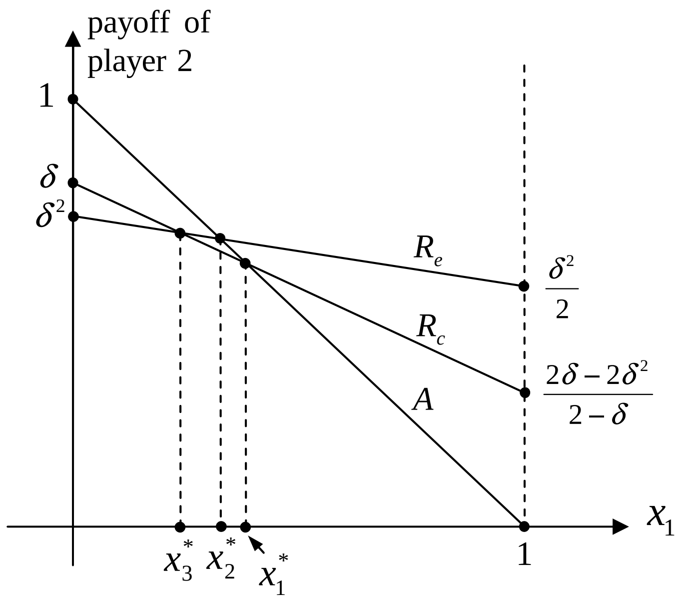


Figure 6: Case 1.

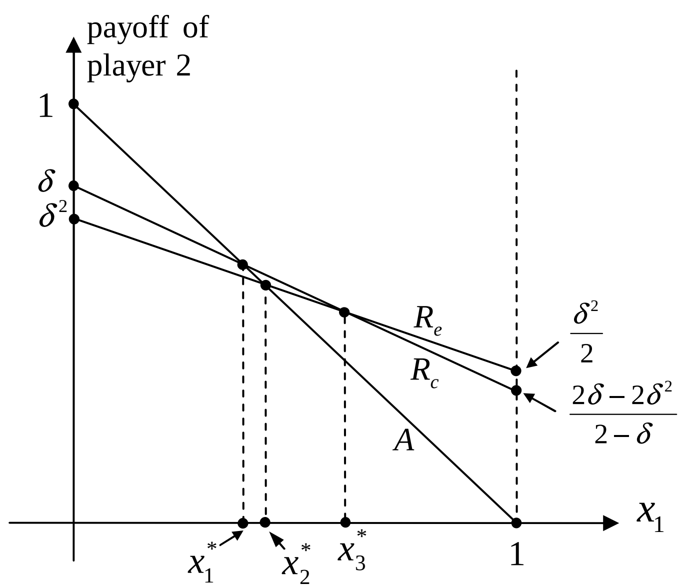


Figure 7: Case 2.

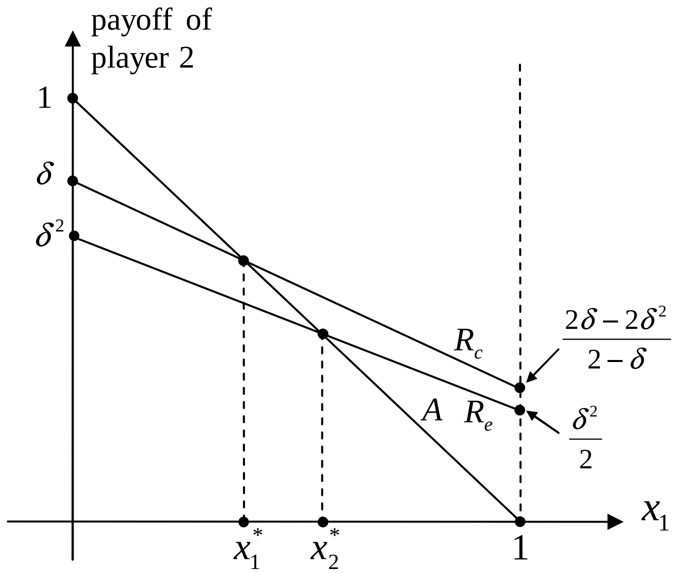


Figure 8: Case 3.

In this case, if Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \leq x_1^*$, then Player 1 will choose option A. If Player 1 makes an offer $(x_1, y_1) \in PF$ with $x_1 \geq x_1^*$, then Player 1 will choose option R_c. □

Proof of Theorem 5:

Using Table 1, one can show that $EP_1(\delta) = \dfrac{2-\delta}{2+\delta}$ and $EP_2(\delta) = \dfrac{2\delta}{2+\delta}$ for $\delta \in [0.781, 0.868]$, $EP_1(\delta) = \dfrac{2-2\delta^2}{2-\delta^2}$ and $EP_2(\delta) = \dfrac{\delta^2}{2-\delta^2}$ for $\delta \in (0.868, 0.874]$, and $EP_1(\delta) = EP_2(\delta) = \dfrac{\delta^2}{2}$ for $\delta \in (0.874, 1)$. It is easy to verify that $EP_1(\delta) \leq EP_2(\delta)$ for any $\delta \in [0.781, 1)$. □

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